Privacy and Polarization: An Inference-Based Framework*

Tommaso Bondi
Cornell Tech

Omid Rafieian
Cornell Tech

Yunfei (Jesse) Yao
The Chinese University of Hong Kong

February 25, 2025

Abstract

Advances in behavioral targeting allow media firms to better monetize based on advertising. However, behavioral ad targeting requires consumer tracking, which has heightened privacy concerns among consumers and regulators. We examine how stricter privacy regulations that ban consumer tracking affect media firms' content strategies and ideological positioning. We consider a model where media firms choose their ideological positioning and advertising, while ideologically heterogeneous consumers select their preferred content based on both their ideology and idiosyncratic taste shocks. We compare two salient informational environments: (1) behavioral ad targeting, where perfect inference about consumers is allowed, and (2) contextual ad targeting, where consumer tracking is banned due to privacy regulations, and media firms can only infer consumer types based on their media choice. We show that privacy regulations that ban behavioral ad targeting incentivize media firms to shift toward more extreme and polarizing positioning in order to draw better inferences about consumer types, even though the shift to more extreme ideological positions can hurt both demand and consumer welfare from content consumption. Compared to the monopoly case, competition increases firms' inference motives and leads to more polarized content over a wider range of parameters due to an inferential complementarity effect arising from consumer self-selection. Our research uncovers a previously unexplored relationship between privacy and polarization, shedding light on the potential unintended consequences of privacy regulations in media markets.

JEL Codes: M37, L82, L13, D83.
Keywords: advertising, targeting, privacy, polarization.

*The author names are listed in alphabetical order. We gratefully acknowledge financial support from the NET Institute at NYU Stern. We also wish to thank Ron Berman, Alessandro Bonatti, Luis Cabral, Sachin Gupta, Ganesh Iyer, Justin Johnson, Vrinda Kadiyali, Dmitri Kuksov, Nanda Kumar, Jura Liaukonyte, Ruslan Momot, Amin Sayedi, Miguel Villas-Boas, Jingbo Wang, and three anonymous ACM Economics and Computation 2024 referees for their valuable comments, as well as seminar participants at Cornell, and conference participants at Marketing Science 2023, UT Dallas Bass FORMS 2024, and Summer Institute in Competitive Strategy 2024. All errors remain our own.

1 Introduction

Digital publishers increasingly use advertising as a monetization strategy. At the core of ad-based monetization is behavioral ad targeting that creates a sustainable revenue stream for publishers and keeps online content mostly free. However, behavioral ad targeting requires the collection and use of consumer-level data, which leads to privacy concerns among consumers. According to a recent survey by Pew Research, over 80% of US adults are concerned about how companies use the data they collect from them, making the need for privacy regulation an issue with bipartisan support (McClain et al., 2023). In response to consumers' privacy concerns in the US and globally, regulatory bodies and even some private firms have started taking actions to restrict consumer tracking online and protect consumer privacy. A few prominent examples include the California Consumer Privacy Act (CCPA) and the European Union's General Data Protection Regulation (GDPR) in the public sector, and Apple's App Tracking Transparency (ATT) and Google's Privacy Sandbox in the private sector.

Although the main intent of privacy regulations is to safeguard consumer privacy, a consistent finding from past empirical research is that privacy regulation hurts digital publishers (Goldfarb and Tucker, 2011; Alcobendas et al., 2021; Johnson et al., 2023). Specifically, prior research suggests that the revenue loss due to privacy regulations is more pronounced for general interest (vs. specialized) publishers, who have greater uncertainty about their consumer types without consumer tracking (Goldfarb and Tucker, 2011). For example, in the absence of consumer tracking, a mainstream news website such as the Associated Press has a harder time inferring consumer types and interests in order to show them relevant ads, compared to a niche, ideologically extreme website such as Fox News, which has more precise information about their consumers. Thus, the negative impact of privacy regulation on news publishers and media firms largely depends on their ideological positioning and content strategies.

The content-dependent impact of privacy regulation on media firms gives rise to an important question: does privacy regulation affect media firms' ideological positioning and content strategies? To counter the loss imposed by privacy regulations, a media firm can shift from a mainstream positioning and content strategy (e.g., general news coverage) to a more specific and niche posi-

tioning and content strategy (e.g., polarizing opinion programs), thereby drawing more accurate inferences about consumers who choose their content and showing them more relevant ads. Such a shift in news content has important implications in the media landscape, especially given the rise in political polarization and media bias. Nevertheless, the extant research on privacy has focused on measuring the impact of regulations on media firms' market *outcomes* and largely ignored their equilibrium response in terms of positioning and content strategies. Focusing on media firms' equilibrium positioning strategies is particularly important given the empirical evidence that show media firms change their ideological positioning over time (Kim et al., 2022).[1]

In this paper, we bridge this gap and endogenize media firms' positioning in markets with and without privacy regulations that ban consumer tracking. We explicitly model the choice of mainstream and niche positioning using a simple Hotelling model of firm positioning and consumer demand. Specifically, we seek to answer the following questions:

1. How does privacy regulation affect the positioning and content strategies employed by media firms who monetize based on advertising in monopoly and duopoly settings?

2. What are the implications in terms of polarization? When do media firms have a greater incentive to shift to a more polarizing positioning?

3. What are the implications in terms of demand, consumers' consumption utility, and media firms' profits?

To answer these questions, we develop a model in which profit-maximizing media firms choose their ideological positioning and advertising, while ideologically heterogeneous readers consume the media content based on both its match with their ideological preferences and their own idiosyncratic taste shocks. In our model, media firms monetize by selling ad impressions to advertisers. As such, their profits are the product of two separate components: (1) an extensive margin that captures the *quantity of impressions*, and (2) an intensive margin that captures the *quality of impressions*.

To study the impact of privacy regulations that ban consumer tracking (e.g., GDPR), we compare two different information environments: *behavioral ad targeting* and *contextual ad targeting*.

[1] We thank one of the anonymous reviewers for suggesting this reference.

The difference between the two comes from the possibility of tracking. Under *behavioral ad targeting*, media firms possess perfect knowledge of each consumer's type because they are allowed to track consumers. The *contextual ad targeting* scenario mimics the situation in the presence of privacy regulations that ban consumer tracking, where media firms are only allowed to use contextual information to target their ads to consumers. The comparison between contextual and behavioral ad targeting allows us to examine the downstream impact of privacy regulations on media firms' equilibrium choices and overall market outcomes.

Under behavioral ad targeting, since media firms can perfectly match ads to consumers on a one-to-one basis, they can achieve maximum quality of impressions regardless of consumers' media choice. As a result, the media firm's profit-maximization problem is greatly simplified. Because perfect ads-to-readers matching is possible, the profit-maximizing positioning is the one that maximizes the quantity of impressions.

Conversely, under contextual ad targeting, media firms no longer have access to perfect information about consumer types, so they need to rely solely on a single piece of information to infer consumer types: consumer's self-selection into media. In this case, media firms have the incentive to deviate from the demand-maximizing positioning (the equilibrium under behavioral ad targeting) and shift to positioning and content strategies that increase the quality of impressions. In other words, media firms may benefit from creating content with lower demand but sharper signals about consumers, thereby achieving higher profits by balancing the quantity and quality of impressions.

We first consider a monopolist media firm and examine its equilibrium strategies under both behavioral and contextual ad targeting regimes. We show that deviating from the demand-maximizing positioning and content strategy can be an equilibrium under contextual ad targeting when consumers are sufficiently ideologically differentiated and when their sensitivity to imperfect ad targeting (mismatches between ad and consumer type) is sufficiently high. In particular, we find that the deviation is more toward extreme and polarizing ideological positioning because media firms can sharpen the signal about consumer types by moving toward the extreme ends of consumers' ideological preferences. Notably, this incentive persists even when such a shift toward extreme content reduces both consumer welfare from content consumption and total demand.

We then study the duopoly case to examine the equilibrium outcomes under the two targeting regimes in a competitive environment. Studying the duopoly case is important because competition naturally incentivizes media firms to differentiate and move towards more polarizing positions. As such, it is not clear whether the inferential motives bolstered by privacy regulation are strong enough to shift the equilibrium to further polarization under contextual ad targeting compared to behavioral ad targeting, as in the monopoly case. Our equilibrium analysis in the duopoly case reveals important insights.

First, we find that the media firms' ideological positioning in the equilibrium under contextual ad targeting is at least as polarizing as that under behavioral ad targeting, which establishes the positive link between privacy regulation and polarization in a competitive environment. Second, compared to the monopoly setting, we document that privacy regulation leads to more polarization even when consumers are less ideologically differentiated and when their sensitivity to imperfect ad targeting is lower. A key underlying mechanism we identify is an *inferential complementarity* effect between the two firms, which indicates that the presence of one firm aids the other firm's inference, making the inferential motives stronger for the media firms. Therefore, inferential complementarity, combined with regular product differentiation incentives under competition, strengthens the shift to more polarization due to privacy regulation under duopoly.

In summary, our paper makes several contributions to the literature. We study the link between privacy and polarization and highlight the possibility of an unintended consequence of privacy regulations in increasing content polarization. Specifically, we show that privacy regulation can lead to greater content polarization, even when it reduces total demand and consumer welfare from content consumption. Our finding is important as it goes against extensive media speculations and policy memos that cite hyper-personalization of online content and the lack of privacy regulation as a key contributor to the increased polarization over the past few decades (Pariser, 2011). The well-documented mechanism is that lack of privacy leads to the possibility of more granular personalization, which creates echo chambers where consumers are only exposed to confirming views. Therefore, less privacy induces more polarization. In our model, we provide new insights into this problem: it is the *inability to personalize* that leads firms to move to more extreme and polarizing

ideological positioning to sharpen their inferences about consumers. Thus, privacy regulation can lead to higher content polarization than cases with perfect content personalization. A key innovation of our framework is in explicitly modeling privacy as an inference problem. In particular, our inference-based framework suggests that by limiting what firms *know* about consumers, privacy regulations shift firms' focus to what they *want to know* about consumers, which creates an incentive to use content polarization as a device to draw more accurate inference. Our framework extends the literature on media bias and complements the existing mechanism for the relationship between privacy and polarization by introducing inference motives as an important determinant of actions chosen by strategic players, which is largely ignored in the prior literature on media markets: notably, we show that privacy leads firms to polarize *beyond* a mere reflection of the underlying ideological partisanship among consumers; thus, the average consumer is exposed to *more ideologically extreme* content than he would be under perfect content personalization. Finally, our framework offers broader implications for the theoretical and empirical literature on privacy. From a game theoretical standpoint, this modeling framework allows us to examine players' equilibrium responses to the privacy shifts. For empirical studies, this provides a framework to quantify the magnitude of information gain and loss in various data environments.

2 Literature Review

Our work relates to the literature on media markets. Early theoretical work in this domain considers the competition between broadcasters in the presence of the advertising market and shares insights into equilibrium outcomes in this market in terms of content provision and advertising strategies (Dukes and Gal-Or, 2003; Gal-Or and Dukes, 2003; Anderson and Coate, 2005; Godes et al., 2009). A separate stream of work in this literature has examined content strategies as they relate to media bias and polarization (Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2006; Xiang and Sarvary, 2007). With the growth of digital news consumption, a series of recent studies have focused on the specific aspects of the digital context and examined pricing and content strategies by media markets (Johar et al., 2012; Sun and Zhu, 2013; Ambrus et al., 2016; Athey et al., 2018; Berman et al., 2019; Lin, 2020; Amaldoss et al., 2021; Jain and Qian, 2021; Amaldoss et al., 2023; Amaldoss

and Du, 2023; Ke et al., 2023). Our paper adds to this stream of work by studying a key aspect of the digital context: behavioral ad targeting and the possibility of privacy regulations. In particular, we study the impact of privacy regulations on media markets and examine the equilibrium outcomes in terms of ideological positioning and content strategies and their implications for media bias and polarization. The transition from behavioral targeting to contextual targeting due to privacy regulations is largely the reverse of the process of the initial adoption of behavioral targeting in the late 1990s (Novatiq, 2024). The key difference between the two transitions is that the advertising choice was much less personalized following the introduction of behavioral tracking than it was before the launch of privacy regulations because real-time bidding was introduced in 2009 (Sweeney, 2023) Therefore, the trade-off between quantity and quality of impressions is a unique feature of the second transition that this paper studies.

Our work relates to the literature on behavioral targeting, personalization, and, more broadly, online advertising. With the advancements in targeting technologies in advertising markets, a series of papers have studied the impact of targeting accuracy or advertising strategy on equilibrium market outcomes (Chen et al., 2001; Iyer et al., 2005; Levin and Milgrom, 2010; Tucker and Zhang, 2010; Bergemann and Bonatti, 2011; Mayzlin and Shin, 2011; Zhang and Katona, 2012; Kuksov et al., 2013; Kamada and Kojima, 2014; Amaldoss et al., 2016; Sayedi, 2018; Choi and Sayedi, 2019; Berman and Katona, 2020; Rafieian and Yoganarasimhan, 2021; Shin and Yu, 2021; Lauga et al., 2022; Berman et al., 2023; Chaimanowong et al., 2023; Choi and Sayedi, 2023; Ning et al., 2023; Shin and Shin, 2023; Yang et al., 2023; Fainmesser et al., 2023a; Choi and Sayedi, 2024; Johnson et al., 2024). In the news media context, many have speculated that greater content personalization results in more polarization, citing the positive correlation between the rise of political polarization in the US and the surge in personalized content delivery through online platforms (Pariser, 2011). Despite the widespread lay belief that causally connects personalization and polarization, empirical findings in this domain do not present a consistent viewpoint. For instance, the demographic groups in the US that are least likely to use the Internet experienced the greatest increase in polarization (Boxell et al., 2017). Moreover, studies analyzing users' browsing histories reveal that, despite social media and the Internet being associated with greater ideological divergence

among users, they also increase exposure to opposing views (Flaxman et al., 2016). Investigations focused on specific platforms and their personalized features offer conflicting results concerning the link between personalization and polarization. Notably, studies investigating Facebook's news feed algorithm, Google's search personalization, and YouTube have found limited evidence suggesting that personalization contributes to content bias (Bakshy et al., 2015; Ribeiro et al., 2020; Hosseinmardi et al., 2021). In our paper, we build on the theoretical literature on ad targeting and develop a model to study the impact of ad personalization on the supply of polarizing content. Our work extends this literature by providing an inference-based theoretical account that presents a more nuanced view of the link between personalization and polarization. In particular, we find that the content produced under no personalization can become *more extreme and polarized* than consumers' preexisting ideological preferences. Therefore, we highlight a case where the content under full personalization is less polarized than that under no personalization.

Our paper also relates to the literature on privacy. A vast body of theoretical work has examined different issues related to consumer identification, privacy, and information markets (Villas-Boas, 1999, 2004; Taylor, 2004; Acquisti and Varian, 2005; Bergemann and Bonatti, 2015; Bergemann et al., 2018; Choi et al., 2020; Rhodes and Zhou, 2021; Choi and Jerath, 2022; Hu et al., 2022; Iyer and Singh, 2022; Momot and Salikhov, 2022; Yang, 2022; Bonatti et al., 2023; Choi et al., 2023; Fainmesser et al., 2023b; Ke and Sudhir, 2023; Lei et al., 2023; Strack and Yang, 2023; Miklós-Thal et al., 2023; Yao, 2024). Empirical papers in this domain have studied the impact of privacy regulations on market outcomes in different settings (Goldfarb and Tucker, 2011; Johnson, 2022; Johnson et al., 2023). In particular, Goldfarb and Tucker (2011) study a change in tracking and targeting regulations and document lower response rates to ads and, therefore, lower ad revenues for publishers. Notably, they demonstrate a heterogeneous effect of privacy regulation on digital publishers, with general interest publishers such as the New York Times experiencing higher revenue loss than specialized publishers such as Car and Driver Magazine. We extend this literature by endogenizing media firms' (publishers) ideological positioning and content strategies, to allow them to respond optimally to the change in privacy policies. We present a generic theoretical framework that characterizes privacy as an inference problem, in which stricter privacy policies have a negative

impact on the accuracy of inference about consumers. Importantly, we identify the possibility of increased polarization as an unintended consequence of privacy regulations in digital markets.

3 Model

We theoretically characterize a market where media firms (e.g., news publishers) create content for consumers and monetize by placing ads. To reflect consumers' ideological preferences, we assume three discrete consumer types $\{0, 1/2, 1\}$, where 0 and 1 refer to the opposing ideological ends (e.g., left vs. right) and $1/2$ refers to the centrist position. Consistent with consumer preferences, $\{0, 1/2, 1\}$ defines the action set for the media firm's ideological positioning or slant, which allows us to characterize the match between the consumer type and the media firm's content. The media firm's ideological positioning informs its general content strategies. As such, we refer to this decision with terms such as ideological positioning, content strategy, and slant interchangeably throughout the paper. While politics is a natural application for our model, our framework encompasses broader settings with ideological preferences about issues (e.g., animal rights vs. hunting websites). We unify our concept by referring to the positioning in the middle ($= 1/2$) as *mainstream* and the positioning at the two extremes ($\in \{0, 1\}$) as *niche*.[2]

We denote the consumer type by θ. We assume that the consumer type has a symmetric distribution that depends on a single parameter λ, with $1-2\lambda$ proportion of users having a centrist or mainstream position ($\theta = 1/2$), and λ proportion of consumers being at each extreme or niche end of consumer types ($\theta \in \{0, 1\}$). As such, we can view λ as a measure of consumer polarization and ideological partisanship. Since we want to study media firms' incentive to polarize in equilibrium with and without privacy regulations, we focus on cases where mainstream consumers are in the minority: $\lambda \geq 1/3$. It is clear that cases with low levels of consumer polarization will lead to minimal content polarization regardless of the targeting regime. In contrast, our focal cases where $\lambda \in [1/3, 1/2]$ represent settings where content polarization can be viewed as a major possibility.[3]

[2]Our use of "niche" and "mainstream" is in line with Johnson and Myatt (2006), and needs not have a market share interpretation.

[3]It is worth noting that our qualitative insight holds for $\lambda < 1/3$. The reason we focus on $\lambda \in [1/3, 1/2]$ is that the insights from these cases have clearer implications for polarization.

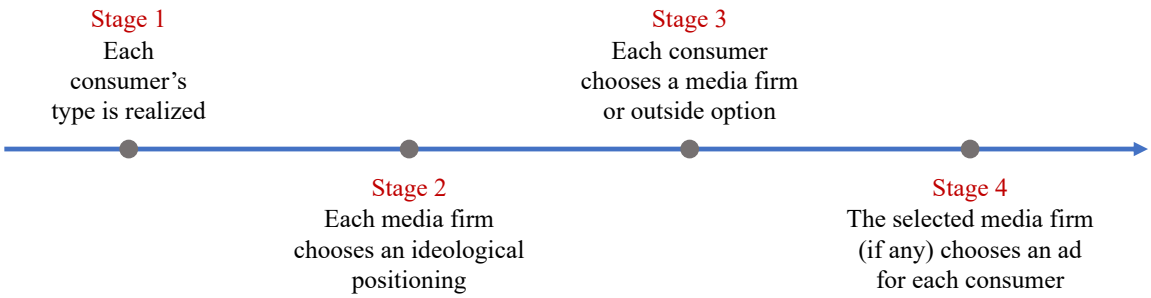


Figure 1: Timeline of the game

We denote the media firm's ideological positioning choice by x, which comes from $\{0, 1/2, 1\}$. If the consumer chooses the media firm, an impression will be generated and the media firm can place an ad $a \in [0, 1]$ in that impression catering to different consumer types. As such, although the ideological positioning x is the same for all consumers, ad a can be targeted at the impression level depending on the information available about the consumer.[4]

We consider two types of information environment, *behavioral ad targeting* and *contextual ad targeting*. The information environment is characterized by the observability of each consumer's type, which depends on whether behavioral tracking and targeting are allowed. When *behavioral ad targeting* is allowed, we assume that the media firm knows the consumer type for each consumer on its platform, as it has rich prior information about the consumer through tracking. On the other hand, when behavioral ad targeting is banned due to privacy regulations, the media firm needs to engage in *contextual ad targeting* and infer consumer type based on the context, e.g., the content consumed by the consumer.

Figure 1 presents the timeline of our game in four stages. We present the details of each stage as follows:

1. In the first stage, consumer type θ is realized. Without privacy regulation (under behavioral ad targeting), it is known by the media firm(s). With privacy regulation (under contextual

[4]Even though the mainstream-type consumer is the minority, the demand for mainstream content can be more than that for niche content under both monopoly and duopoly. This is because the consumer faces some idiosyncratic shock in content consumption (the error term of the utility function in Equation (1) on the next page). On average, a consumer is farther from a niche position than a mainstream one. So, mainstream content attracts more consumers of a different type than niche content does. As we will show in §5, the equilibrium content strategy may be 1/2 in the monopoly case and (1/2, 1/2) in the duopoly case.

ad targeting), it is the consumer's private information.

2. In the second stage, each media firm chooses an ideological positioning strategy x to maximize its expected profits. The media firm's choice x informs its content strategy. The media firm can choose among one mainstream position at $x = 1/2$ and two niche positions at $x = 0$ and $x = 1$. In the political context, the niche positions can be interpreted as left- and right-leaning ideological positions. The media firm's positioning choice is broadcasted to all consumers through their content. Consistent with the literature on media slant, the ideological positioning or slanting strategy is a single choice, so the firm cannot be right- and left-leaning at the same time (Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2010).[5]

3. In the third stage, each consumer consumes at most one content unit.[6] We define a type-θ consumer's utility from consuming the content by the media firm with ideological positioning x as follows:

$$U(x;\theta) = 1/2 - |x - \theta| + \epsilon, \tag{1}$$

where the first term is the base utility from media consumption that is set to $1/2$, the second term is the consumer's distaste for ideological mismatch as measured by the distance of the media firm's ideological positioning from the consumer type, and the third term is an idiosyncratic error term that is firm-specific with mean zero that comes from the distribution $f(\cdot)$ such that $f(\epsilon) > 0,\ \forall \epsilon \in [-1/2, 1/2]$. Conceptually, this idiosyncratic error term captures components of a consumer's utility from consuming content by a media firm that is unrelated to their ideology θ, such as a desire to be informed or a preference for content variety.[7] Intuitively, a higher $Var(\epsilon)$ reflects a greater openness by the consumer to ideologically mismatched content. We can generalize the base utility to any v (see §6.1), but fixing

[5]The intuitive reason behind this restriction is the reputational cost that the firm would face by holding multiple ideological positions at once. It is worth emphasizing that we do not focus on news aggregators as they are only distributors of content, not creators.

[6]The assumption that only one content unit is consumed is without loss of generality. As long as individual consumption does not depend on θ, all of our findings remain unchanged.

[7]The inclusion of the idiosyncratic term is justified by two important empirical observations. First, there is substantial evidence that consumers switch between media firms different in ideological positions (Gentzkow and Shapiro, 2011). Second, without a stochastic component, the choice of media firm fully reveals the consumer type, making behavioral and contextual ad targeting the same, which runs counter to most empirical findings that behavioral ad targeting yield substantially higher revenues (Goldfarb and Tucker, 2011; Johnson, 2022).

it at $v = 1/2$ conveys the main insights while keeping the model simple. We normalize the utility from the consumer's outside option to 0. For tie-breaking rules, we assume that the consumer randomly visits one firm with equal probability if multiple firms give the consumer the same utility, and that the consumer visits the firm if she is indifferent between consuming the media content and choosing the outside option.

The utility framework in Equation (1) implies that a consumer located at one extreme ($\theta = 0$ or 1) will never consume the content by a media firm located at the other extreme ($x = 1$ or 0), but may consume the content by a mainstream media firm ($x = 1/2$) if her idiosyncratic term is sufficiently high.[8] Mainstream consumers ($\theta = 1/2$) may consume the mainstream or either niche media firms, depending on the relative values of the idiosyncratic terms.

4. In the final stage, the media firm chooses ad a if the consumer generates an impression by choosing the media firm's content. The expected value generated from showing ad a to a consumer of type θ is given by the match value function $\mathcal{M}$ as follows:

$$\mathcal{M}(a;\theta) = 1 - \gamma(a-\theta)^2, \tag{2}$$

where the match value convexly decreases in the mismatch between ad and consumer $|a-\theta|$, and the parameter γ captures the *targeting value* and is an instrumental parameter for our analysis.[9] It is worth emphasizing that although the media firm chooses the ad, having this choice is not a requirement in our framework and this choice can be delegated to ad intermediaries (e.g., Google) who run auctions among advertisers to allocate ad impressions. We demonstrate the equivalence of our modeling framework with one that allocates ads in a competitive auction marketplace in Online Appendix 1.

[8] In the general base utility extension in §6.1, the consumer located at one extreme may consume content from the opposite extreme and the main insights are robust.

[9] We remark that this formulation essentially reduces consumer types to one dimensional or, in other words, assumes that ideological and advertising preferences are perfectly correlated and abstract away from ads that are not ideological. While this is a simplification motivated by analytical convenience, our modeling framework maps well into settings where ad targeting is based on the corresponding ideology. The prime example of such contexts is political advertising (e.g., donations) where the ad value depends on the political ideology of consumers. However, the modeling framework extends to other cases where such tribal segments exist. To that end, a more polarized consumption pattern–as shown in Schoenmueller et al. (2023)–creates more value in ideology-based targeting, although we acknowledge that the correlation, while positive, is not perfect in those cases.

We assume that the match value function $\mathcal{M}(\cdot\,;\cdot)$ is a continuous and smooth function in both a and θ. As we can see from Equation (2), the maximum value of the match value function is one. We do not restrict ourselves to the case of $\gamma \leq 1$; therefore, a large ad-consumer mismatch ($(a-\theta)^2 > 1/\gamma$) can result in negative ad profits. These could be interpreted, for instance, as the ad being repulsive to consumers (e.g., a hunting rifle ad for a vegetarian consumer), alienating them in the future.

We now define the media firm's profit maximization problem. Let $D(x)$ denote the total consumer demand for a media firm at position x. This is the *quantity* of impressions the media firm is able to generate. For each impression, the firm can place an ad and obtain ad revenue that depends on the *quality* of the impression characterized by the match value function. We assume that there is a perfectly competitive ad marketplace, so the revenue-per-impression is the same as the expected match value of the ad for a given impression.[10] Let $\mathcal{I}$ denote the information available for each impression. We define the firm's optimal profits π given information $\mathcal{I}$ as follows:

$$\pi(x, a; \mathcal{I}) = \max_{x,a} D(x) \cdot \mathbf{E}\left[\mathcal{M}(a;\theta) \mid \mathcal{I}\right], \tag{3}$$

where the firm jointly maximizes content strategy x and ad a given the information $\mathcal{I}$ available.[11] The firm's contextual information about an impression is that the consumer has chosen the media firm's content x. In any event, the firm has contextual information about an impression, so we have $x \in \mathcal{I}$. Under behavioral ad targeting, we assume that the firm also has information about the exact consumer type θ. Since we are interested in cases under *behavioral* and *contextual* ad targeting, we characterize the information available under each case as follows:

- The information under *behavioral* ad targeting is defined as $\mathcal{I}_b$ and includes the contextual

[10] The prior literature suggests the possibility of thin markets under behavioral ad targeting (Levin and Milgrom, 2010; Zhang and Katona, 2012; Rafieian and Yoganarasimhan, 2021). However, the assumption of a perfectly competitive ad market is reasonable in our context because we have three broad consumer types. As such, if the market is thick enough for each, our assumption holds. Further, even in the presence of a thin market, the media firm can still extract a higher revenue from a better ad match by setting a reservation price, which is fundamentally the assumption we make.

[11] Although we consider the case where ad allocation happens after consumer's content choice to be more consistent with the real practice of online ad allocation, it is important to emphasize that none of our results would change if we advertising decisions are made after the ideological positioning decision but before consumers' content choices, as long as these decisions are made for each possible scenario in $\mathcal{I}$.

information x as well as behavioral information θ, i.e., $\mathcal{I}_b = \{x, \theta\}$.

- The information under *contextual* ad targeting is defined as $\mathcal{I}_c$ and includes the contextual information x, i.e., $\mathcal{I}_c = \{x\}$.

We specifically model the information under behavioral and contextual ad targeting as extreme cases to obtain cleaner results and better understand the incentives under these two information environments. However, our framework is flexible, and one could easily consider more middle-ground cases where there is more contextual information or less behavioral information. For example, instead of a complete ban, regulations such as GDPR allow users to opt in for behavioral targeting. In such cases, firms have information about some consumers even under privacy regulations. Moreover, the probability of opting in for behavioral targeting may be correlated with user type. If mainstream consumers are more likely to opt in, then a mainstream firm will face a higher uncertainty about consumer type, while a niche firm will face a lower uncertainty about consumer type among consumers who do not opt in when a privacy regulation is launched. This will strengthen a firm's inference motives and, thereby, our result. In contrast, if niche consumers are more likely to opt in, then a mainstream firm will face a lower uncertainty about consumer type, while a niche firm will face a higher uncertainty about consumer type among consumers who do not opt in when a privacy regulation is launched. This will weaken a firm's inference motives and, thereby, our result. Empirically, most consumers do not opt in for behavioral tracking when the consent dialog is explicit and clear(Laziuk, 2021; Goldberg et al., 2024). So, the impact of considering the user's choice of privacy terms on the main result is small.

4 Consumer Privacy and Firm's Inference Problem

A natural way to think about consumer privacy in our setting is to examine how uncertain the firm is about the consumer's type given the information $\mathcal{I}$. We present our definition of consumer privacy as follows:

Definition 1 (Consumer Privacy)**.** *The consumer's privacy given the information $\mathcal{I}$ is defined as* Privacy($\mathcal{I}$) *and is equal to the conditional variance of consumer type given information $\mathcal{I}$, that is,*

$\text{Privacy}(\mathcal{I}) = \mathbf{Var}(\theta|\mathcal{I})$.[12]

We can easily verify that the consumer's privacy is equal to zero under behavioral ad targeting, as $\mathcal{I}_b$ fully reveals the consumer's type. On the other hand, the consumer's privacy under contextual ad targeting depends on the consumer's content/media choice. Some choices can reveal more about the consumer type, thereby reducing the posterior variance of type and consumer privacy. This is an important consideration for the media firm when choosing the ideological positioning and content strategy. We characterize the relationship between consumer privacy and content choice in the following definition:

Definition 2 (Privacy Reducing Choice)**.** *Consumer's content choice x is privacy-reducing over content choice x' if $Privacy(\{x\}) < Privacy(\{x'\})$.*

This definition reflects the insight that a consumer who chooses Fox News or New York Times reveals more about their type than a consumer who chooses Associated Press. To see how the privacy measure enters the media firm's problem, we revisit the firm's profit maximization problem as presented in Equation (3). Since the ad choice is the final decision in the timeline, we start with the firm's optimal ad choice. For ad choice a, the firm needs to infer the consumer type given information $\mathcal{I}$. The following lemma characterizes the media firm's optimal ad choice:

Lemma 1. *The firm's optimal ad choice a^* is the posterior mean of consumer type given information $\mathcal{I}$, i.e., $a^* = \mathbf{E}[\theta \mid \mathcal{I}]$.*

This lemma follows from the property of the variance and the quadratic specification of $\mathcal{M}$.[13] It indicates that the optimal ad a^* is the same as consumer type θ under behavioral ad targeting: $a^*(\theta) = \theta$, while it is equal to the mean of θ given the consumer's content choice x under contextual ad targeting: $a^*(x) = \mathbf{E}(\theta|x)$. Given the ad choice characterized in Lemma 1, we can establish the following result about the media firm's optimal ideological positioning as follows:

[12] Our definition of privacy is similar to the existing measures of privacy, such as differential privacy (Dwork et al., 2014), in capturing the uncertainty about an individual. However, differential privacy measures generally provide the definition over a database and "seeks to guarantee to the potential participant that, irrespective of her participation decision, almost the same things can be learned from the released outcome," thereby granting individuals plausible deniability and offering researchers a worst-case guarantee (Heffetz and Ligett, 2014). Because our work is not concerned with the identifiability of individual records from a database, we use a natural variance-based definition.

[13] In other words, with a different functional form for $\mathcal{M}(a, \theta)$, the firm might find it optimal to deviate from $a^* = \mathbf{E}[\theta \mid \mathcal{I}]$ in the contextual ad targeting case.

Corollary 1. *The media firm chooses the ideological positioning and content strategy x that maximizes $D(x) \cdot [1 - \gamma \text{Privacy}(\mathcal{I})]$.*

The corollary implies that the firm's profits are negatively correlated with consumer privacy. Under behavioral ad targeting, the firm fully observes consumer type (i.e., $Privacy(\mathcal{I}_b) = 0$), so the firm chooses the ideological positioning that maximizes its total demand:

$$\pi^b(x) = \max_x D(x), \tag{4}$$

where $\pi^b(x)$ is shorthand for the profits under behavioral ad targeting, and the firm is only interested in maximizing the *quantity* of impressions. Under contextual ad targeting, the firm needs to infer consumer type given the consumer's self-selection into the media firm's content, so the profit maximization problem simplifies to:

$$\pi^c(x) = \max_x D(x)[1 - \gamma Privacy(\{x\})], \tag{5}$$

where $\pi^c(x)$ is shorthand for the profits under contextual ad targeting, and $Privacy(x)$ is the privacy of consumer type under contextual ad targeting where the only information available about the consumer is their content choice x. As shown in Equation (5), the media firm has to trade off the *quantity* of impressions with the *quality* of impressions. As a consequence, the demand-maximizing ideological positioning is not necessarily the optimal one; the firm may trade off more popular positioning with a more accurate inference about the consumer. In our equilibrium analysis, we examine this inference incentive as it relates to polarization.

5 Equilibrium

5.1 Equilibrium Concept

Due to the setup of a multi-stage game with incomplete information, we consider pure strategy Perfect Bayesian Equilibrium. Specifically, when the firm chooses the advertising location in the last period, it uses Bayes' rule to update its belief about consumer type based on the consumer's

equilibrium content choice.

5.2 Monopoly

5.2.1 Demand

When there is only one firm, the consumer compares the utility of consuming the monopoly's content with the utility of choosing the outside option. Thus, she will consume the content if and only if the utility from content consumption is positive. The following lemma calculates consumer demand given different content choices.

Lemma 2. *The total demand for a monopolist media firm depends on their ideological positioning strategy as follows:*

(a) The total demand for a monopoly's mainstream positioning, $x = 1/2$, is $1 - 2F(0)\lambda$. Among them, $1 - 2\lambda$ are mainstream consumers, $[1 - F(0)]\lambda$ are type 0 consumers, and $[1 - F(0)]\lambda$ are type 1 consumers.

(b) The total demand for a monopoly's niche positioning, $x = 0$ or 1, is $1 - F(0) + [2F(0) - 1]\lambda$. Among them, λ are type $\theta = x$ consumers and $[1 - F(0)](1 - 2\lambda)$ are mainstream consumers.

This lemma highlights an important, general, and natural property. On average, a random consumer is farther from a niche position than a mainstream position. Hence, if the media firm chooses the mainstream ideological positioning, $x = 1/2$, both types of niche consumers, $\theta \in \{0, 1\}$, may consume it because they are not located too far away from the content. In contrast, if the firm chooses the niche ideological positioning, say $x = 0$, mainstream consumers may consume it, while type $\theta = 1$ consumers will not, independent of their idiosyncratic taste shocks, because they are too ideologically distant from the content. This endogenous selection leads to an asymmetry in the firm's inference ability. By choosing niche ideological positioning $x = 0$, the firm knows for sure that it does not attract any type $\theta = 1$ consumers, and can better infer the consumer's type by Bayesian updating.

5.2.2 Media Firm's Ideological Positioning

The monopolist's positioning strategy is straightforward under behavioral ad targeting. It chooses the ideological positioning that maximizes total demand because there is no inference problem, and profit simply equals total demand, as indicated in Equation (4). The following proposition characterizes the monopolist equilibrium under behavioral ad targeting:

Proposition 1. *Under behavioral ad targeting, the monopolist media firm chooses the mainstream ideological positioning if* $[4F(0)-1]\lambda < F(0)$ *and niche ideological positioning if* $[4F(0)-1]\lambda > F(0)$.

This proposition characterizes the equilibrium strategy under the general case of behavioral ad targeting. The optimality condition for the mainstream positioning can easily be satisfied. For example, if we make the common assumption that the distribution of the idiosyncratic term is symmetric, we find that the mainstream positioning is always the equilibrium strategy. This is because $F(0) = 1/2$ under a symmetric distribution, which implies that for all possible values of λ, mainstream positioning is the equilibrium strategy.

As discussed earlier, the firm's perfect inference under behavioral ad targeting implies that the firm does not strategically consider inference. In contrast, under contextual ad targeting, it relies on Bayesian updating to infer consumer type and considers inference strategically. As such, it may have an incentive to deviate from the demand-maximizing strategy (i.e., equilibrium under behavioral ad targeting) and choose an ideological positioning that allows more accurate ad targeting. Hence, the question is under which strategy the firm knows more about consumer type in an impression. The following lemma helps characterize the decision-making trade-off for the media firm:

Lemma 3. *The consumer's choice of niche content is privacy-reducing over the choice of mainstream content, i.e.,* $\text{Privacy}(\{1/2\}) > \text{Privacy}(\{x\})$*, such that* $x \in \{0, 1\}$.

This lemma indicates that a mainstream positioning leads to a lower intensive margin (ad revenue) on each impression, because the firm will have greater uncertainty about the consumer type given their content choice. Therefore, there is a natural incentive for media firms to move to more niche ideological positions. If the niche positioning is demand-maximizing and thus the equilibrium under behavioral ad targeting (when $[4F(0) - 1]\lambda > F(0)$), Lemma 3 implies that it

will be the equilibrium under contextual ad targeting, because niche strategy does better on both elements in the expected profit equation in Equation (5): *quantity* and *quality* of impressions.

When the demand-maximizing strategy is mainstream ideological positioning, the trade-off between quantity and quality of impressions becomes more interesting. Naturally, we expect the equilibrium to depend on the value of targeting that is captured by the parameter γ. The following proposition characterizes this relationship:

Proposition 2. *Under contextual ad targeting, the monopolist media firm's equilibrium strategy depends on the values of λ and γ as follows:*

(a) If $[4F(0) - 1]\lambda > F(0)$, the media firm chooses niche ideological positioning in equilibrium.

(b) If $[4F(0)-1]\lambda < F(0)$, there exists $\gamma^m > 0$ for any value of λ such that the media firm chooses niche ideological positioning if $\gamma > \gamma^m$ and mainstream ideological positioning if $\gamma < \gamma^m$. The threshold γ^m is decreasing in λ.

Figure 2 combines the equilibrium under behavioral and contextual ad targeting and presents three regions. We find that equilibrium strategies are only different in region B (shaded) where $[4F(0)-1]\lambda < F(0)$ and $\gamma > \gamma^m$. In this case, the monopolist media firm chooses a niche ideological positioning in equilibrium under contextual ad targeting, but a mainstream positioning under behavioral ad targeting. Although the media firm generates fewer impressions by choosing a niche ideological positioning, it faces lower uncertainty about consumer type in these impressions. This lower uncertainty translates into a higher revenue-per-impression, which makes up for the lower demand when the value of targeting is sufficiently high, i.e., $\gamma > \gamma^m$. Together, the ideological positioning that the media firm chooses under contextual ad targeting is at least as polarizing as that under behavioral ad targeting.

From a privacy standpoint, our analysis shows that a privacy regulation that bans consumer tracking can lead to increased polarization and partisanship in the supply side. Intuitively, this is because consumer tracking better separates the media firm's ideological positioning choice from its advertising choice. Moreover, our results shed light on the well-studied link between consumers' ideological partisanship (demand) and content polarization (supply), where the prior literature finds

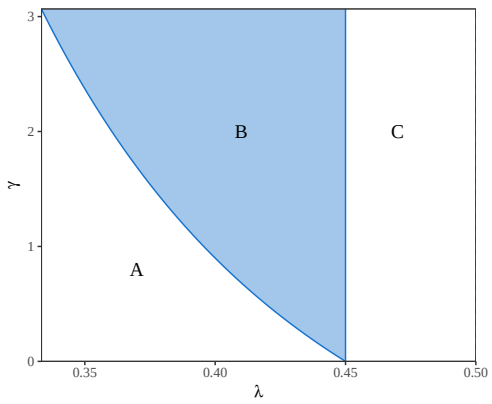


Figure 2: Equilibrium regions in monopoly under behavioral and contextual ad targeting

Note: This figure shows equilibrium regions when $F(0) = 9/16$. Region A shows the cases where mainstream positioning is the equilibrium under both targeting regimes. Region B (shaded) illustrates cases where privacy regulation leads to higher polarization because the firm chooses niche positioning under contextual ad targeting, but a mainstream positioning under behavioral ad targeting. Region C shows cases where the firm chooses niche positioning under both targeting regimes.

a natural incentive for media firms to slant their content (Gentzkow and Shapiro, 2010). We revisit this link with and without privacy regulations and find that when consumers are more ideologically partisan (higher λ), privacy regulation can change the media firm's equilibrium strategy from moderate (mainstream) to partisan (niche) for a wider range of γ, because the threshold γ^m is decreasing in λ. This is a natural comparative static: when more consumers have strong ideological positions, the media firm is more likely to pander to them, even for lower values of targeting. Thus, we find that privacy regulation strengthens the link between consumers' ideological partisanship (demand) and content polarization (supply).

5.2.3 Consumer Utility from Content Consumption

We have shown that the monopolist media firm may trade off the total demand for a more accurate inference when behavioral tracking is banned. In this section, we ask the following question: how does the switch from mainstream to niche content strategy affect consumers' utility from content consumption? Intuitively, niche consumers receive a higher utility from consuming niche content, while mainstream consumers receive a higher utility from consuming mainstream content. Hence, niche content strategy leads to lower consumer utility from content consumption if most consumers

are mainstream (low λ), while it leads to higher consumer utility from content consumption if most consumers are niche (high λ). We formalize this insight in the following proposition:

Proposition 3. *Niche content strategy leads to lower consumer utility from content consumption if and only if* $\lambda < \hat{\lambda} := \{1 - 2[1 - F(0)]\mathbf{E}[\epsilon|\epsilon > 0]\}/\{3 - 8[1 - F(0)]\mathbf{E}[\epsilon|\epsilon > 0]\}$.

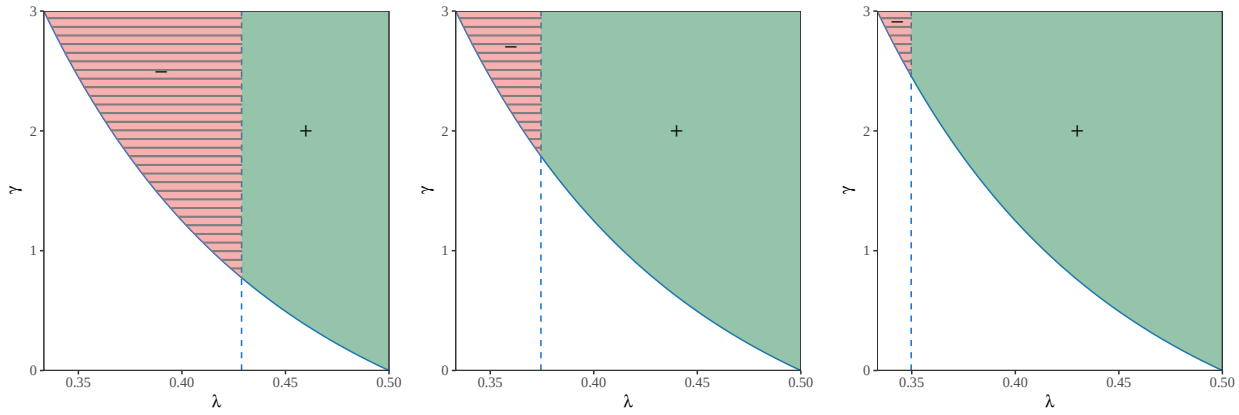
We can now combine this proposition with Propositions 1 and 2 and examine when the equilibrium strategy under contextual ad targeting leads to higher or lower consumer utility from content consumption, compared to the equilibrium under behavioral ad targeting. This is an important comparison from a privacy policy standpoint because it helps us understand a key welfare consequence of privacy regulations. To illustrate this aspect, we need to focus on where the equilibrium strategies are different with and without privacy regulation ($[4F(0) - 1]\lambda < F(0)$ and $\gamma > \gamma^m$) and examine under what condition privacy regulation has unintended consequences in terms of consumer welfare from content consumption. The following corollary highlights this insight:

Corollary 2. *Consumer utility from content consumption is lower under contextual ad targeting than under behavioral ad targeting if* $\lambda < \hat{\lambda}$, $[4F(0) - 1]\lambda < F(0)$, *and* $\gamma > \gamma^m$.

The cutoff threshold $\hat{\lambda}$ determines the region where privacy regulation has an unintended consequence in terms of consumer welfare. This cutoff depends critically on the distribution of the idiosyncratic term. In particular, $\hat{\lambda}$ is increasing in $\mathbf{E}(\epsilon|\epsilon > 0)$.[14] Therefore, understanding $\mathbf{E}(\epsilon|\epsilon > 0)$ is essential to the interpretation of our results.

In general, a distribution of ϵ that is concentrated around 0 results in low values of $\mathbf{E}(\epsilon|\epsilon > 0)$, while the opposite is true for a bimodal distribution that concentrates mass near the extremes $-1/2$ and $1/2$, for which $\mathbf{E}(\epsilon|\epsilon > 0) \to 1/2$. As such, $\mathbf{E}(\epsilon|\epsilon > 0)$ has a close relationship with the variance of the idiosyncratic term ϵ, which captures consumers' media preferences that are unexplained by ideology. In that sense, the higher the variance of the idiosyncratic term, the more open the consumer is to consuming content farther from their own ideology. This is an important construct because many behavioral nudges to encourage people to read from other sides attempt to reduce the relative importance of ideology in consumer utility.

[14] The first derivative is $2[1 - F(0)]/(8[1 - F(0)]\mathbf{E}(\epsilon|\epsilon > 0) - 3)^2$, which is always non-negative.



(a) Bimodal ϵ:
$\epsilon \sim Beta(0.1, 0.1) - 1/2$

(b) Uniform ϵ:
$\epsilon \sim Beta(1, 1) - 1/2$

(c) Bell-shaped ϵ:
$\epsilon \sim Beta(10, 10) - 1/2$

Figure 3: Impact of privacy regulation on consumer utility from content consumption.
Note: The green shaded region with the plus sign shows the region where consumer utility from content consumption is higher under contextual ad targeting than behavioral ad targeting. The red striped region with the minus sign shows the region where the equilibrium under contextual ad targeting leads to lower consumer utility from content consumption. The blank region shows the area where the consumer utility is the same under both behavioral and contextual ad targeting.

Interestingly, our results suggest that as consumers' openness to ideologically different content increases (higher variance of ϵ), the region where privacy regulation reduces consumer welfare from content consumption grows (higher $\hat{\lambda}$). Focusing on the two extreme cases is illuminating. First, if there is no variance in the idiosyncratic term ($\epsilon = 0$), the cutoff $\hat{\lambda}$ will be equal to $1/3$, implying that the privacy regulation always increases consumer welfare whenever it changes the equilibrium under behavioral ad targeting. In contrast, in the limit case of a bimodal ϵ where $\mathbf{E}(\epsilon|\epsilon > 0) \to 1/2$, we have $\hat{\lambda} = 1/2$, which indicates that the equilibrium shift caused by privacy regulation reduces consumer welfare from content consumption. We illustrate this point in Figure 3, where we use three different error distributions by varying the shape parameters of the Beta distribution, upon normalizing its expected value to 0. As shown in each of the three subfigures in Figure 3, as the relative importance of ideology in consumer preferences decreases through changing the variance of the idiosyncratic term, the region that reduces consumer utility from content consumption expands.

In sum, our analysis shows several important welfare implications. First, we show that there always exists a parameter range such that privacy regulation leads to a lower consumer utility from

content consumption.[15] This implies that banning behavioral tracking may drive the monopolist media firm to switch from mainstream to niche ideological positioning, even when doing so reduces both total demand and consumer welfare from content consumption. Second, we show that the negative impact on consumer welfare from content consumption is more pronounced when ideology plays a smaller role in determining consumers' media preferences, a case that is often the target of depolarization efforts.

Before concluding our study of the monopoly case, we emphasize that the negative consequence of privacy regulation highlighted in Proposition 3 and Corollary 2 depends solely on consumers' utility from content consumption, and does not account for additional utility such as consumer's disutility from polarization, from decreased privacy, and from potentially being exposed to offensive (that is, heavily mismatched) ads. In addition, the social welfare may suffer from a higher level of polarization. Because the impact of privacy regulation on polarization and on the consumer's utility from content consumption is of sufficient importance and interest, we abstract away from the above considerations in this paper. Due to the above reasons, increasing content polarization per se may be viewed as an unintended consequence of privacy regulations. In a richer model where one takes into account the additional utility terms, privacy regulation can hurt consumer or social welfare by increasing the level of polarization and decreasing consumer privacy even if its direct impact on consumer's utility from content consumption is non-negative.

5.3 Duopoly

We now introduce competition in a duopoly setting. The impact of competition in our setting is both *a priori* unclear and conceptually interesting: are the two firms' inference motives complements or substitutes? Further, given that competition generally leads to an increase in differentiation, is it still true that contextual ad targeting leads to *more* content polarization compared to behavioral ad targeting? Moreover, provided that an increase in differentiation occurs following a ban on individual data tracking, what are its implications for consumer demand and utility from media consumption? To answer these questions, we consider a duopoly setting in this section. Note

[15] We show in the proof of Proposition 3 that $\hat{\lambda} > 1/3$ for any distribution of the idiosyncratic term ϵ.

that when there are multiple firms at the same position, the consumer's idiosyncratic term ϵ for each firm's content is drawn independently from the distribution $f(\cdot)$, which captures a taste shock for consuming content by each media firm.

5.3.1 Demand

In contrast to the monopoly case, consumers compare the utility of consuming a firm's content not only with the outside option but also with the utility of consuming the other firm's content. Suppose that the first media firm chooses x_1 ideological positioning, while the second one chooses x_2 ideological positioning. Let (x_1, x_2) denote firms' positioning strategies. Without loss of generality, we assume that $x_1 \leq x_2$. By symmetry, (0,0) is equivalent to (1,1), and (0,1/2) is equivalent to (1/2,1). Hence, we only need to consider (0,0), (0,1/2), (0,1), and (1/2,1/2). One can see that (0,1) dominates (0,0), so (0,0) will never be an equilibrium. Hence, we need to only consider three possible equilibria: (0,1), (1/2,1/2) and (0,1/2). Lemma 4 fully characterizes consumer demand under each possible equilibrium strategies for the two firms as follows:

Lemma 4. *The media firms' demand under three possible equilibria (0,1), (1/2,1/2), and (0,1/2) is characterized as follows:*

(a) Under (0,1) strategy profile, Firm 1's total demand is $\lambda + [1 - F(0)^2](1 - 2\lambda)/2$. *Among them,* λ *are type 0 consumers and* $[1 - F(0)^2](1 - 2\lambda)/2$ *are mainstream consumers. Firm 2's total demand is* $\lambda + [1 - F(0)^2](1 - 2\lambda)/2$. *Among them,* λ *are type 1 consumers and* $[1 - F(0)^2](1 - 2\lambda)/2$ *are mainstream consumers.*

(b) Under (1/2,1/2) strategy profile, each firm's total demand is $(1-2\lambda)/2+[1-F(0)^2]\lambda$. *Among them,* $(1 - 2\lambda)/2$ *are mainstream consumers,* $[1 - F(0)^2]\lambda/2$ *are type 0 consumers, and* $[1 - F(0)^2]\lambda/2$ *are type 1 consumers.*

(c) Under (0,1/2) strategy profile, Firm 1's total demand is $\lambda[1 - \int_{-1/2}^{0} F(\epsilon_0)f(\epsilon + 1/2)d\epsilon] + (1 - 2\lambda)\int_{-1/2}^{0} F(\epsilon)f(\epsilon + 1/2)d\epsilon$. *Among them,* $\lambda[1 - \int_{-1/2}^{0} F(\epsilon)f(\epsilon + 1/2)d\epsilon]$ *are type 0 consumers and* $(1 - 2\lambda)\int_{-1/2}^{0} F(\epsilon)f(\epsilon + 1/2)d\epsilon$ *are mainstream consumers. Firm 2's total demand is* $\lambda\int_{-1/2}^{0} F(\epsilon)f(\epsilon + 1/2)d\epsilon + (1 - 2\lambda)[1 - \int_{-1/2}^{0} F(\epsilon)f(\epsilon + 1/2)d\epsilon] + [1 - F(0)]\lambda$. *Among them,*

$\lambda \int_{-1/2}^{0} F(\epsilon) f(\epsilon + 1/2) d\epsilon$ *are type 0 consumers,* $(1 - 2\lambda)[1 - \int_{-1/2}^{0} F(\epsilon) f(\epsilon + 1/2) d\epsilon]$ *are type 1/2 consumers, and* $[1 - F(0)]\lambda$ *are type 1 consumers.*

Even without inference motivation (i.e., in the case of behavioral ad targeting), competing firms have an incentive to choose a more polarized content strategy to increase differentiation and soften the competition, which increases the total demand. Therefore, it is, in some sense, harder for them to choose a *more* polarized content strategy (compared to the baseline level of differentiation) when the information environment shifts from behavioral ad targeting to contextual ad targeting. Nevertheless, the next section shows that the inference motivation becomes stronger with competition.

5.3.2 Media Firms' Ideological Positioning

For tractability, we make an additional assumption that $\epsilon \sim U[-1/2, 1/2]$, which simplifies the demand function characterized in Lemma 4. The following propositions summarize the equilibria under behavioral and contextual ad targeting.

Proposition 4. *Suppose* $\epsilon \sim U[-1/2, 1/2]$. *Under behavioral ad targeting, the equilibrium is (1/2,1/2) if* $\lambda < \lambda_1$, *(1/2,1/2) or (0,1) if* $\lambda_1 < \lambda < \lambda_2$, *and (0,1) if* $\lambda > \lambda_2$, *where* $1/3 < \lambda_1 = 4/11 < \lambda_2 = 3/7$.

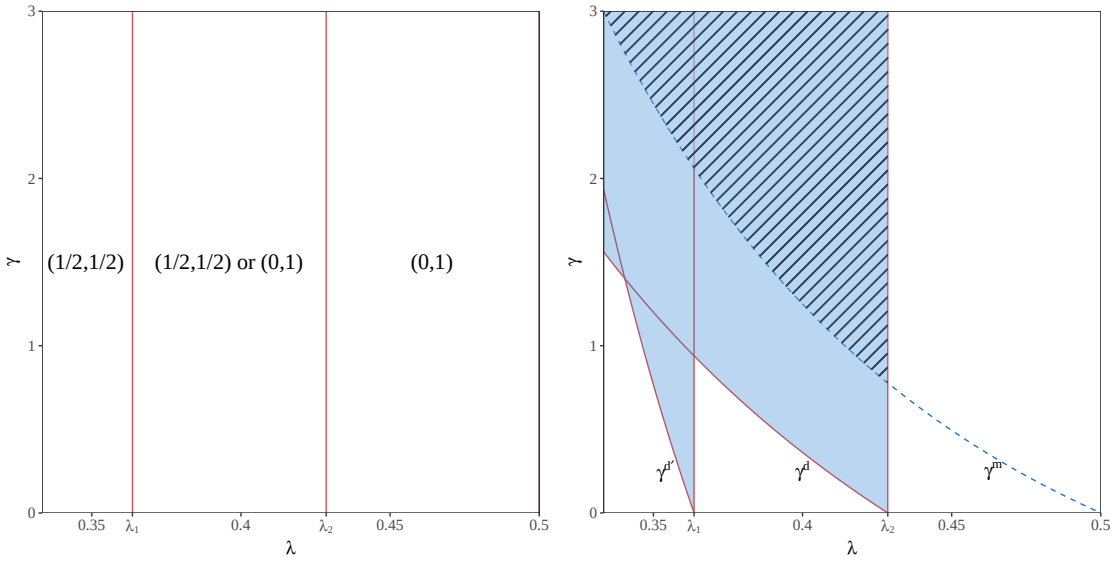
Proposition 5. *Suppose* $\epsilon \sim U[-1/2, 1/2]$. *The equilibria are (weakly) more polarized under contextual ad targeting than under behavioral ad targeting.*[16] *The following table summarizes cases where the media firms choose more polarized content strategies under contextual ad targeting than under behavioral ad targeting, where* $1/3 < \lambda_0 < \lambda_1$, $\gamma^d > 0$ *for* $\lambda < \lambda_2$, *and* $\gamma^{d'} > 0$ *for* $\lambda < \lambda_1$.[17]

[16] The extent of media polarization increases from (1/2,1/2) to (0,1/2) to (0,1).

[17] Table 1 in the appendix presents the equilibria for all the cases.

Condition	*Equilibrium Under*	
	Behavioral Ad Targeting	*Contextual Ad Targeting*
$1/3 < \lambda < \lambda_0$ & $\gamma^d < \gamma < \gamma^{d'}$	*(1/2,1/2)*	*(0,1/2)*
$\lambda_0 < \lambda < \lambda_1$ & $\gamma^{d'} < \gamma < \gamma^d$	*(1/2,1/2)*	*(1/2,1/2) or (0,1)*
$1/3 < \lambda < \lambda_1$ & $\gamma > \max\{\gamma^d, \gamma^{d'}\}$	*(1/2,1/2)*	*(0,1)*
$\lambda_1 < \lambda < \lambda_2$ & $\gamma > \gamma^d$	*(1/2,1/2) or (0,1)*	*(0,1)*

Figure 5 illustrates media firms' content strategies in duopoly under behavioral (left figure) and contextual (right figure) ad targeting. The content strategies are the same under both behavioral and contextual ad targeting in the blank region, while the extent of media polarization is larger under contextual ad targeting than under behavioral ad targeting in the solid and striped-shaded regions.



(a) Behavioral Ad Targeting (b) Contextual Ad Targeting

Figure 4: Media firms' equilibrium content strategies in duopoly when $\epsilon \sim U[-1/2, 1/2]$.
Note: The blank region shows areas where the equilibrium is the same under behavioral and contextual ad targeting in duopoly. The shaded area shows the total area where the equilibrium is more polarized under contextual ad targeting than behavioral ad targeting. The striped region shows the part of the shaded area where the equilibrium under contextual ad targeting is more polarizing than the equilibrium under behavioral ad targeting in monopoly too.

When $\lambda > \lambda_2$, there are enough niche consumers (at least 6/7 of the total) that (0,1) is the equilibrium in duopoly even under behavioral ad targeting. As such, there is no room for the

duopoly to be more polarized if the advertising environment switches from behavioral ad targeting to contextual ad targeting. Now consider the interesting case in which $\lambda < \lambda_2$, so that at least $1/7$ of consumers are mainstream. In both monopoly and duopoly, contextual ad targeting leads to more content polarization when $\gamma > \gamma^m$ (the striped shaded region in Figure 4b). In addition, contextual ad targeting leads to more polarization when $\min\{\gamma^d, \gamma^{d'}\} < \gamma < \gamma^m$ and $\lambda < \lambda_1$ or when $\gamma^d < \gamma < \gamma^m$ and $\lambda_1 < \lambda < \lambda_2$ (the solid shaded region in Figure 4b) under duopoly but not under monopoly. In particular, when $1/3 < \lambda < \lambda_1$ and $\gamma > \max\{\gamma^d, \gamma^{d'}\}$, equilibria across the two regimes display opposite properties: equilibrium under behavioral ad targeting features both firms choosing the mainstream ideological positioning, while equilibrium under contextual ad targeting features the two firms picking opposite niche ideological positions. Thus, privacy regulation leads firms to switch from "underproviding" to "overproviding" (compared to the underlying consumer ideology distribution) highly partisan positions.

Therefore, even *conditional on differentiation motives due to competition*, duopolistic firms have a *stronger* incentive than a monopoly to polarize due to inference motives. In other words, compared to a monopolist, duopolistic firms choose a more polarized strategy under contextual rather than behavioral ad targeting in a wider range of parameters, as long as the competition is not strong enough that they already have chosen the most polarized strategy under behavioral ad targeting.

The underlying mechanism is the following. On the one hand, competing media firms directly cannibalize each other's demand if they choose the mainstream content strategy. In contrast, they can cover different niche consumers by choosing the opposite ideological positions and thereby soften the competition. As a result, the benefit of higher total demand in the market by choosing mainstream content strategy is lower in the duopoly case than in the monopoly case. On the other hand, the benefit of a more accurate inference is not affected by competition - a monopolistic firm and a duopolistic firm extract the same amount of surplus in the advertising market for a given level of consumer privacy. In addition, by choosing niche ideological positioning, a duopolistic firm can obtain a more accurate inference than a monopoly due to an *inferential complementarity* effect. The presence of one firm aids the other firm's inference because competition improves the selection of consumers: on average, consumers of each firm are more ideologically aligned with that firm,

compared to the monopoly case. For instance, in the $(0, 1)$ case, both firms lose relatively more demand from consumers of opposite ideologies, thus decreasing the variance in consumer types for each firm and increasing profits per impression in the contextual ad targeting regime.[18] Therefore, firms lean more toward quality in the quantity-quality trade-off under a duopoly than under a monopoly.

The fact that an increase in privacy results in an increase in equilibrium content polarization is interesting in light of the well-discussed link between content personalization (that is, lack of privacy) and polarization. It is important to stress that, according to this view, personalized content does not *polarize* consumers; rather, it simply *matches* their prior ideological positions. In this case, one could say that content positioning is the *consequence*, rather than the *cause*, of polarization.

The drivers of Proposition 5 are quite different: the desire to make precise inferences to increase advertising profits pushes two competing firms away from the standard Hotelling equilibrium, $(1/2, 1/2)$, and toward $(0, 1)$ that is more ideologically polarized than the underlying distribution of the consumers' ideological preferences, for any value of $\lambda < 1/2$. Therefore, more privacy leads to more content polarization on the supply side. Content polarization is driven by firms' desire to monetize ads, not just by consumer ideology; in fact, content distribution often becomes more polarized than consumers.

5.3.3 Consumer Utility from Content Consumption

As we have shown in the monopoly case, a more polarized content strategy may lead to lower consumer utility from content consumption. However, with two firms, polarization becomes desirable for consumers. The next proposition shows that consumer utility from content consumption always increases in the extent of polarization under duopoly.

Proposition 6. *For any $\lambda \in (1/3, 1/2)$ and $\epsilon \sim U[-1/2, 1/2]$, a more polarizing equilibrium in duopoly leads to a higher consumer utility from consumption: (0,1) leads to higher consumer utility*

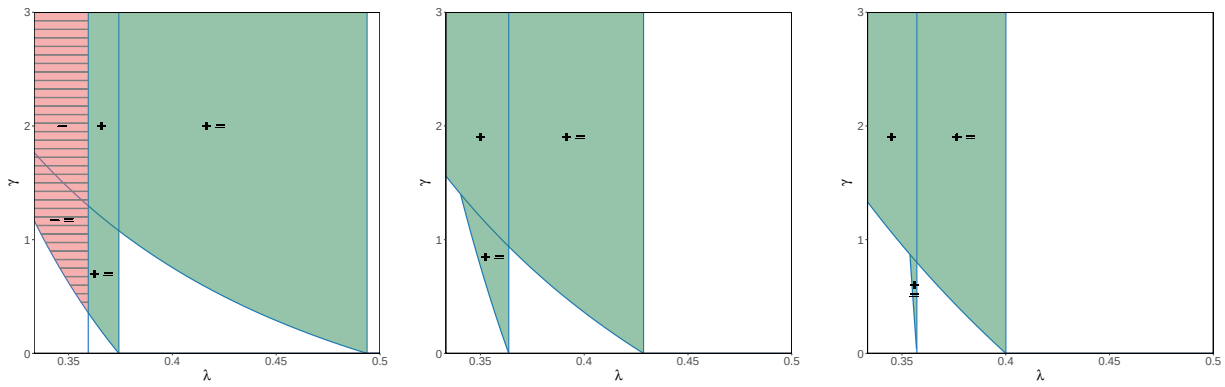
[18] This complementarity motive arises as long as at least one firm chooses niche content. When both firms choose mainstream strategies, $(1/2, 1/2)$, each firm's demand is simply cut in half, and their inferential ability is the same as a monopoly, as no additional consumer ideological self-selection arises.

from content consumption than (0,1/2), and (0,1/2) leads to higher consumer utility from content consumption than (1/2,1/2).

This proposition helps us examine the implications of a privacy regulation that enforces a move from behavioral to contextual ad targeting. As shown in Proposition 5, the equilibrium under contextual ad targeting is at least as polarizing as the equilibrium under behavioral ad targeting for the entire parameter space. Thus, we can write the following corollary:

Corollary 3. *For any* $\lambda \in (1/3, 1/2)$ *and* $\epsilon \sim U[-1/2, 1/2]$*, privacy regulation leads to higher consumer utility from content consumption in a duopoly.*

This corollary shows that a privacy regulation in the duopoly context does not have unintended consequences in terms of the content consumption aspect of consumer welfare when ϵ is uniformly distributed. This is different from our findings in the monopoly setting, which shows that the shift to greater content polarization under privacy regulation can reduce consumers' utility from content consumption under uniform distribution. Intuitively, our findings in the duopoly setting indicate that consumers enjoy higher content polarization, which is expected given that we consider cases where consumers are polarized to begin with ($\lambda > 1/3$). We further run numerical simulations for other distributions of the idiosyncratic term, $\epsilon \sim Beta(0.1, 0.1) - 1/2$ and $\epsilon \sim Beta(10, 10) - 1/2$ and present the results in Figure 5. Compared to Figure 3 in the monopoly case, we find similar patterns that as consumers' content choice becomes less revealing of their ad content preference (higher variance of ϵ), the region where privacy regulation reduces consumer welfare from content consumption grows. Notably, we replicate the finding under monopoly that the region where privacy regulation reduces consumer utility from content consumption grows as consumers' openness to ideologically different content increases (higher variance of ϵ). In addition, conditional on the variance of the idiosyncratic term, privacy regulation hurts consumers' utility from content consumption less when there is competition. In summary, we find that compared to the monopoly case, in a competitive environment, privacy regulation helps consumers receive higher utility from content consumption, although at the expense of an increase in content polarization.



(a) Bimodal ϵ:
$\epsilon \sim Beta(0.1, 0.1) - 1/2$

(b) Uniform ϵ:
$\epsilon \sim Beta(1, 1) - 1/2$

(c) Bell-shaped ϵ:
$\epsilon \sim Beta(10, 10) - 1/2$

Figure 5: Impact of privacy regulation on consumer utility from content consumption (duopoly case).
Note: The green shaded region with the plus sign shows the region where consumer utility from content consumption is always higher under contextual ad targeting than behavioral ad targeting. The green shaded region with the plus-equal sign shows the region where consumer utility from content consumption is higher under contextual ad targeting than behavioral ad targeting in some equilibria and identical in other equilibria. The red striped region with the minus sign shows the region where the equilibrium under contextual ad targeting leads to lower consumer utility from content consumption. The red striped region with the minus-equal sign shows the region where the equilibrium under contextual ad targeting leads to lower consumer utility from content consumption under some equilibria and leads to identical consumer utility from content consumption under other equilibria. The blank region shows the area where the consumer utility is the same under both behavioral and contextual ad targeting.

6 Extensions

6.1 General Base Utility

Throughout our exposition so far, the consumer's base utility from media consumption was fixed at 1/2. This allows us to convey all the main insights about the inference role of content strategy and the quantity-quality trade-off while keeping the model simple. Nevertheless, it is interesting to study whether and how the incentives to opt for mainstream or niche positioning differ for firms of different quality. In this extension, we generalize the base utility to v, and show the robustness of our findings. In addition, we demonstrate how the size of the base utility strengthens/weakens the firm's inference incentive.

Proposition 7. *Suppose* $\epsilon \sim U[-1/2, 1/2]$. *There exists* $\underline{v} \leq 7/25, \overline{v} \geq 5/6, \gamma^m > 0, \gamma^{d''}, \lambda_1 < \lambda_2$ *and* $\overline{\lambda}(v) \in (1/3, 1/2]$ *such that the following holds if* $\underline{v} < v < \overline{v}$*:*

(a) (Monopoly) The firm chooses mainstream content under behavioral ad targeting and niche

content under contextual ad targeting if $1/3 < \lambda < \overline{\lambda}(v)$ *and* $\gamma > \gamma^m$. *The threshold* γ^m *decreases in* λ *and increases in* v. *In addition, there exists* $\hat{\lambda}(v) > 1/3$ *such that niche content strategy leads to lower consumer welfare if* $\lambda < \hat{\lambda}(v)$.

(b) (Duopoly) The equilibrium under behavioral ad targeting is (1/2,1/2) if $\lambda < \lambda_1$, *(1/2,1/2) or (0,1) if* $\lambda_1 < \lambda < \lambda_2$, *and (0,1) if* $\lambda > \lambda_2$. *The equilibrium under contextual ad targeting is more polarized than that under behavioral ad targeting if* $\gamma > \gamma^{d''}$ *and* $1/3 < \lambda < \lambda_2$. *Moreover,* $\gamma^{d''} < \gamma^m$ *for all* $\lambda \in (1/3, \lambda_2)$.

As we can see, for an interval range of the base utility that includes 1/2, we recover all the main insights from Propositions 1 to 5 in the main model. It is still the case that the media firm's ideological positioning has an inference role for the firm under contextual ad targeting, that a media firm will choose more polarized content in equilibrium to increase profit per consumer despite the lower total demand and potentially lower consumer welfare, and that competition strengthens firms' inference incentive.

In the main model, a consumer located at one extreme will never consume the content located at the other extreme. In contrast, a consumer located at one extreme may consume the opposite content when the base utility v is larger than 1/2. In particular, a type 0 consumer will obtain a positive utility from consuming $x = 1$ content with probability $\mathbf{P}(v - |x - \theta| + \epsilon > 0) = 1/3$ if $v = 5/6$. Therefore, our extension to the general base utility establishes the robustness of our results to settings where a consumer located at one extreme may consume content from the opposite extreme.

The general base utility setup provides additional insight into how the base utility affects the firm's inference incentive. The firm has a smaller incentive to improve its inference by choosing more polarized ideological positioning as the base utility increases because γ^m increases in v. The base utility can be viewed as the vertical quality of the media firm, whereas the match between consumer type and media/ad location is horizontal. When the vertical quality is high, the media firm attracts consumers with ideological preferences far from the firm's position. As a result, the horizontal match quality becomes less important and it becomes harder for the firm to distinguish different consumer types based on their self-selection. The firm is more incentivized to favor quantity in the

quantity-quality trade-off by choosing mainstream content with a larger total demand.

In contrast, when the base utility is low, consumers are more selective in their consumption decisions. Mainstream consumers will not consume niche content that has a low vertical quality and poor horizontal match unless the random shock is very high (a large idiosyncratic term). So, by choosing the more polarized niche content strategy, the firm can screen consumer types very accurately and thus obtain a high profit per consumer by offering well-matched ads to most consumers. In this case, the firm has a stronger incentive to favor quality in the quantity-quality trade-off by choosing a niche content strategy.

In summary, our extension to the general base utility shares important insights into the link between privacy and content polarization. In particular, we find that the impact of privacy regulation on content polarization is stronger when the base utility is lower. On the other hand, the link becomes weaker when the base utility is higher. Thus, policymakers can focus their attention on policies that incentivize firms to produce a higher vertical content quality.

6.2 Consumer Utility from Advertising

In the main model, consumers derive utility solely from media consumption. This assumption can be justified by two arguments. First, it is reasonable to expect that the consumer places an overwhelming majority of the weight on the media content rather than on the advertising content. Second, taking into account the utility of future ads would require the consumer to be forward-looking, which is not always the case.

In this extension, we allow consumers to gain more disutility from worse-matched ads and to take it into account in their content choices.[19] We show that our qualitative results still hold as long as consumers place the majority of the weight on the media content rather than on the advertising content, an assumption that has been empirically validated in recent studies (Brynjolfsson et al., 2024). Formally, we consider the following utility specification:

$$U(x, \theta) = 1/2 - |x - \theta| - \mathbf{E}[f(a, \theta)] + \epsilon,$$

[19] Analogously, we can consider an alternative specification where the consumer gains more utility from better-matched ads, which does not change the results. We chose the current setting because there is more evidence that consumers dislike advertising rather than enjoying seeing an ad.

where a is the advertising choice. Analogously to the rest of our analysis, under behavioral ad targeting the firm offers personalized ads to each consumer, $a = \theta$, whereas under contextual ad targeting the ads can only depend on the media content choice, $a = \mathbf{E}_x(\theta)$, which is the expected consumer type among consumers who consume the media content. We make the following assumptions on f to reflect the idea that the consumer prefers better-matched ads but places the majority of the weight on the media content.

Assumption 1. *The advertising disutility function satisfies the following conditions:*
(1). $f(a, \theta)$ *increases in* $|a - \theta|$; (2). $f(\theta, \theta) = 0$; (3). $f(a, \theta) \leq M$.

In this assumption, M represents an upper bound for ad disutility. The empirical literature on ad avoidance suggests that the advertising volume leads to ad avoidance, but the evidence remains limited on the ad content playing a major role in driving content consumption (Wilbur, 2016; Brynjolfsson et al., 2024). Hence, with the same level of advertising volume, it is reasonable to expect that the upper bound M is small relative to $|x - \theta|$. To understand why we need this assumption, consider an extreme case in which consumers place a very high weight on the advertising disutility term. Suppose the firm chooses niche content $x = 0$. None of the type 0 consumers consume the content, whereas mainstream consumers consume it as long as their idiosyncratic term is positive. Under this unrealistic strategy profile, the optimal ad choice is $1/2$ because only mainstream consumers consume the media content in equilibrium. The expected disutility term is 0 for mainstream consumers but large for niche consumers. The difference in the advertising disutility term more than compensates the difference in the media consumption term, $1/2 - |x - \theta| + \epsilon$. Such strategic implications can overrule our mechanism in the main model.

Under behavioral ad targeting, the equilibrium is the same as the main model because the firm can perfectly infer consumer type and thus show perfectly matched ads. The case of contextual targeting is more complex. The reason for this is that under contextual ad targeting, each consumer's strategy now depends not only on the firm's strategy and her own type but also on the other consumers' actions. The following proposition shows that our main qualitative insight from earlier analysis holds:

Proposition 8. *Suppose* $\epsilon \sim U[-1/2, 1/2]$. *There exists* $\hat{M} > 0$ *such that the consumer's choice of niche content is privacy-reducing over the choice of mainstream content and that all the main insights of the monopoly case hold qualitatively if* $M \leq \hat{M}$.

6.3 General Oligopolistic Case (More Than Two Firms)

In this section, we extend the monopoly and duopoly model to a case with a fixed set of media firms. Consistent with the duopoly case, we consider firm-specific idiosyncratic error terms that are independent and come from $U[-1/2, 1/2]$. Suppose that we have $K > 2$ firms. In Web Appendix 3, we derive the closed-form conditional probabilities of consumer's content choice given type, for any given number of firms at each position (N_0, M, N_1). We note that the signal becomes exponentially sharp from the content choice as the number of firms offering content x increases. In particular, as shown in Web Appendix 3, with L firms at position x, the probability that a consumer of type $\theta = x$ chooses a media firm at another position is upper bounded by $(1/2)^L$. This indicates that with a very large number of firms at each position, the amount of inference firms can make under behavioral and contextual targeting becomes identical. However, since we have the closed-form probabilities, we can computationally examine the equilibrium in cases with more than two firms. Through extensive simulations with $K = 3, 4$ (e.g., Figure 6 in the Web Appendix 3), we establish the same insight: there is no set of parameters under which the equilibrium under behavioral targeting is more polarizing than that under contextual targeting.

6.4 Endogenous Entry

In principle, privacy regulation might influence not just the content strategies of existing firms but also the number of firms operating in equilibrium. In this extension, we endogenize the number of firms, so that the equilibrium number of firms may change following a ban on behavioral ad targeting. To this end, we assume that there is a fixed entry cost $c > 0$ for each firm, and there are enough potential entrants such that new firms will always enter the market as long as they can make a non-negative profit. When the entry cost c is too large, no firm operates in equilibrium, and the problem becomes uninteresting. Further, given that we have studied the monopoly and

duopoly case in our main model, in this extension, we assume that c is small enough such that there is at least one firm at each position $x = 0, 1/2, 1$. In line with our baseline model, we maintain the assumption that each firm can only pick one position.[20]

Although our discrete modeling framework is sufficient for a small number of firms, it does not capture the complexities of a large market. The fundamental reason is that with 3 positions, firms cannot differentiate enough. This implies that the probability of a type-θ consumer consuming content at position $x \neq \theta$ will exponentially decrease with more firms at a position, which in turn implies that contextual targeting will be identical to behavioral targeting, two properties at odds with the empirical reality of media markets. Therefore, we consider a position-specific rather than firm-specific idiosyncratic term in this section.[21] Lastly, we make a technical assumption on the number of firms in this extension: there is a continuum of firms rather than an integer number of firms. This assumption simplifies the analyses and has been adopted in seminal papers studying market entry (Hopenhayn, 1992; Melitz, 2003).[22]

In equilibrium, free entry and market clearing imply that each firm makes zero profits. Following privacy regulation, each consumer becomes less valuable to advertisers because uncertainty about the consumer's type leads to worse ad-matching quality, resulting in a lower total number of firms. Therefore, we will compare equilibrium outcomes with and without privacy regulation by the share of niche firms in each. We have the following result.

Proposition 9. *The consumer's choice of niche content is privacy-reducing over the choice of mainstream content. The ratio of the number of niche firms to the number of mainstream firms is higher under contextual ad targeting than under behavioral ad targeting.*

As the proposition shows, the main insight in the main model extends to the case with endogenous entry when there are more firms and when the equilibrium number of firms depends endogenously on the privacy policy. Put differently, we find that with endogenous entry and pri-

[20] If a firm offers multiple products at different positions, then each product can be viewed as a separate "firm" in this setting.

[21] Our qualitative insight that the equilibrium under contextual ad targeting is at least as polarizing as that under behavioral ad targeting still holds under the position-specific idiosyncratic term in the duopoly case, as in the main model.

[22] Including the integrality constraint in the model does not provide any relevant economic insights.

vacy regulation, mainstream firms represent a smaller slice of a smaller pie. As a result, the decrease in the provision of centrist news is strengthened compared to the exogenous case.

7 Conclusion

The increased use of behavioral ad targeting by websites has heightened concerns by consumer privacy advocates and regulators. In this paper, we study the consequences of privacy regulations in the media landscape and ask the following question: how does a privacy regulation affect the equilibrium strategies of media firms in terms of ideological positioning and content polarization? We theoretically examine this question by building a model of product positioning and consumer demand. In our model, media firms choose their ideological positioning and create content, and consumers select their preferred content based on their ideology and idiosyncratic shocks. Depending on the eyeballs media firms generate, these firms extract advertising revenue based on the expected efficacy of their ads.

To examine the impact of privacy regulations, we focus on two primary information environments: behavioral and contextual ad targeting. Under *behavioral ad targeting*, media firms can perfectly track consumers and target their ads, reflecting the case without privacy regulation. In contrast, under *contextual ad targeting*, media firms cannot track readers and are therefore limited to tailoring ads to content, relying on the information contained in consumers' self-selection into the media firm's content. We show that banning behavioral ad targeting incentivizes media firms to shift towards niche (or polarizing) content strategies to improve their ability to draw inferences about consumers. In monopoly, this holds even when the increase in content polarization decreases total demand and consumer utility from content consumption. In duopoly, we document a stronger shift toward content polarization due to privacy regulation, though this shift does not reduce consumer utility from content consumption.

The findings of this study shed light on the complex connection between privacy and polarization. In recent years, many in the press have underscored the pivotal role played by increasingly precise personalization and behavioral ad targeting in the surge of political polarization within the United States. This perspective posits that hyper-personalization due to the lack of privacy

regulation led to the creation of echo chambers and increased polarization. However, our research presents an alternative reversed link between privacy and content polarization. In our model, the inability to personalize content compels firms to use their content strategy as a targeting strategy that relies on consumers' self-selection into content. This strategic shift creates a unique incentive for producing more partisan content in both monopoly and duopoly scenarios.

Our paper contributes to the literature in several ways. First, we identify a counter-intuitive and theoretically robust link between privacy and polarization. We show that a privacy regulation leads to greater content polarization in equilibrium when media firms monetize based on advertising, even when content polarization reduces total demand and consumer utility from content consumption. Therefore, our paper identifies the firm's inference motive as another driver of content polarization beyond consumer demand and highlights the unintended consequences of privacy regulations in media markets. Second, we bring a novel inference-based framework to study the equilibrium effects of privacy regulation. In particular, our framework suggests that by limiting what firms *know* about consumers, privacy regulations shift firms' focus to what they *want to know* about consumers, thereby creating an incentive to use content strategies as means for more accurate inference. Put differently, we stress an important shift from viewing content polarization as a mere reflection of consumers' preexisting preferences, which is the case with personalization. Under contextual ad targeting, content consumed can become *more extreme and polarized* than consumers' preexisting preferences. Lastly, our paper offers important implications for managers and policymakers. In particular, we present the equilibrium effect of privacy regulations in media markets as a function of interpretable model parameters. We show that privacy regulations can have an unintended consequence of increased content polarization. Further, we propose that investments that increase content quality can mitigate the negative effects of privacy regulations on content polarization.

Nevertheless, our paper has certain limitations. First, we focused on discrete consumer types for tractability and interpretability reasons. Future research can extend our framework to a continuous setting. Second, in order to highlight the media firm's problem, we did not explicitly model the advertising marketplace. One could endogenize the advertising marketplace and obtain new insights for the advertising side of the market. Finally, our paper abstracted away from the dynamics

between content polarization and opinion polarization. Future research could incorporate these dynamics and examine their long-run equilibrium.

References

Alessandro Acquisti and Hal R Varian. Conditioning prices on purchase history. *Marketing Science*, 24(3):367–381, 2005.

Miguel Alcobendas, Shunto Kobayashi, and Matthew Shum. The impact of privacy measures on online advertising markets. *Available at SSRN 3782889*, 2021.

Wilfred Amaldoss and Jinzhao Du. How can publishers collaborate and compete with news aggregators? *Journal of Marketing Research*, 2023.

Wilfred Amaldoss, Kinshuk Jerath, and Amin Sayedi. Keyword management costs and "broad match" in sponsored search advertising. *Marketing Science*, 35(2):259–274, 2016.

Wilfred Amaldoss, Jinzhao Du, and Woochoel Shin. Media platforms' content provision strategies and sources of profits. *Marketing Science*, 40(3):527–547, 2021.

Wilfred Amaldoss, Jinzhao Du, and Woochoel Shin. Pricing strategy of competing media platforms. *Marketing Science*, 2023.

Attila Ambrus, Emilio Calvano, and Markus Reisinger. Either or both competition: A "two-sided" theory of advertising with overlapping viewerships. *American Economic Journal: Microeconomics*, 8(3):189–222, 2016.

Simon P Anderson and Stephen Coate. Market provision of broadcasting: A welfare analysis. *The review of Economic studies*, 72(4):947–972, 2005.

Susan Athey, Emilio Calvano, and Joshua S Gans. The impact of consumer multi-homing on advertising markets and media competition. *Management science*, 64(4):1574–1590, 2018.

Eytan Bakshy, Solomon Messing, and Lada A Adamic. Exposure to ideologically diverse news and opinion on facebook. *Science*, 348(6239):1130–1132, 2015.

Dirk Bergemann and Alessandro Bonatti. Targeting in advertising markets: implications for offline versus online media. *The RAND Journal of Economics*, 42(3):417–443, 2011.

Dirk Bergemann and Alessandro Bonatti. Selling cookies. *American Economic Journal: Microeconomics*, 7(3):259–294, 2015.

Dirk Bergemann, Alessandro Bonatti, and Alex Smolin. The design and price of information. *American economic review*, 108(1):1–48, 2018.

Ron Berman and Zsolt Katona. Curation algorithms and filter bubbles in social networks. *Marketing Science*, 39(2):296–316, 2020.

Ron Berman, Shiri Melumad, Colman Humphrey, and Robert Meyer. A tale of two twitterspheres: Political microblogging during and after the 2016 primary and presidential debates. *Journal of Marketing Research*, 56(6):895–917, 2019.

Ron Berman, Aniko Oery, and Xudong Zheng. Influence or advertise: The role of social learning in influencer marketing. *Available at SSRN 4324888*, 2023.

Alessandro Bonatti, Yunhao Huang, and J Villas-Boas. A theory of the effects of privacy. Technical report, Discussion paper, MIT and UC Berkeley, 2023.

Levi Boxell, Matthew Gentzkow, and Jesse M Shapiro. Greater internet use is not associated with faster growth in political polarization among us demographic groups. *Proceedings of the National Academy of Sciences*, 114(40):10612–10617, 2017.

Erik Brynjolfsson, Avinash Collis, Asad Liaqat, Daley Kutzman, Haritz Garro, Daniel Deisenroth, and Nils Wernerfelt. The consumer welfare effects of online ads: Evidence from a 9-year experiment. Technical report, National Bureau of Economic Research, 2024.

Wee Chaimanowong, Francisco Cisternas, and Stylianos Despotakis. The effects of microtargeting on conversion rates and the role of information asymmetry. *Available at SSRN 4525987*, 2023.

Yuxin Chen, Chakravarthi Narasimhan, and Z John Zhang. Individual marketing with imperfect targetability. *Marketing Science*, 20(1):23–41, 2001.

W Jason Choi and Kinshuk Jerath. Privacy and consumer empowerment in online advertising. *Foundations and Trends® in Marketing*, 15(3):153–212, 2022.

W Jason Choi and Amin Sayedi. Learning in online advertising. *Marketing Science*, 38(4):584–608, 2019.

W Jason Choi and Amin Sayedi. Open and private exchanges in display advertising. *Marketing Science*, 42(3):451–475, 2023.

W Jason Choi and Amin Sayedi. Agency market power and information disclosure in online advertising. *Marketing Science*, 2024.

W Jason Choi, Kinshuk Jerath, and Miklos Sarvary. Advertising and price competition under consumer data privacy choices. *Available at SSRN 3708273*, 2020.

W Jason Choi, Kinshuk Jerath, and Miklos Sarvary. Consumer privacy choices and (un) targeted advertising along the purchase journey. *Journal of Marketing Research*, 60(5):889–907, 2023.

Anthony Dukes and Esther Gal-Or. Negotiations and exclusivity contracts for advertising. *Marketing Science*, 22(2):222–245, 2003.

Cynthia Dwork, Aaron Roth, et al. The algorithmic foundations of differential privacy. *Foundations and Trends® in Theoretical Computer Science*, 9(3–4):211–407, 2014.

Itay P Fainmesser, Andrea Galeotti, and Ruslan Momot. Consumer profiling via information design. *Available at SSRN*, 2023a.

Itay P Fainmesser, Andrea Galeotti, and Ruslan Momot. Digital privacy. *Management Science*, 69 (6):3157–3173, 2023b.

Seth Flaxman, Sharad Goel, and Justin M Rao. Filter bubbles, echo chambers, and online news consumption. *Public opinion quarterly*, 80(S1):298–320, 2016.

Esther Gal-Or and Anthony Dukes. Minimum differentiation in commercial media markets. *Journal of Economics & Management Strategy*, 12(3):291–325, 2003.

Matthew Gentzkow and Jesse M Shapiro. Media bias and reputation. *Journal of political Economy*, 114(2):280–316, 2006.

Matthew Gentzkow and Jesse M Shapiro. What drives media slant? evidence from us daily newspapers. *Econometrica*, 78(1):35–71, 2010.

Matthew Gentzkow and Jesse M Shapiro. Ideological segregation online and offline. *The Quarterly Journal of Economics*, 126(4):1799–1839, 2011.

David Godes, Elie Ofek, and Miklos Sarvary. Content vs. advertising: The impact of competition on media firm strategy. *Marketing Science*, 28(1):20–35, 2009.

Samuel G Goldberg, Garrett A Johnson, and Scott K Shriver. Regulating privacy online: An economic evaluation of the gdpr. *American Economic Journal: Economic Policy*, 16(1):325–358, 2024.

Avi Goldfarb and Catherine E Tucker. Privacy regulation and online advertising. *Management science*, 57(1):57–71, 2011.

Ori Heffetz and Katrina Ligett. Privacy and data-based research. *Journal of Economic Perspectives*, 28(2):75–98, 2014.

Hugo A Hopenhayn. Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, pages 1127–1150, 1992.

Homa Hosseinmardi, Amir Ghasemian, Aaron Clauset, Markus Mobius, David M Rothschild, and Duncan J Watts. Examining the consumption of radical content on youtube. *Proceedings of the National Academy of Sciences*, 118(32), 2021.

Ming Hu, Ruslan Momot, and Jianfu Wang. Privacy management in service systems. *Manufacturing & Service Operations Management*, 24(5):2761–2779, 2022.

Ganesh Iyer and Shubhranshu Singh. Persuasion contest: Disclosing own and rival information. *Marketing Science*, 41(4):682–709, 2022.

Ganesh Iyer, David Soberman, and J Miguel Villas-Boas. The targeting of advertising. *Marketing Science*, 24(3):461–476, 2005.

Sanjay Jain and Kun Qian. Compensating online content producers: A theoretical analysis. *Management Science*, 67(11):7075–7090, 2021.

Monica Johar, Nanda Kumar, and Vijay Mookerjee. Content provision strategies in the presence of content piracy. *Information Systems Research*, 23(3-part-2):960–975, 2012.

Garrett Johnson. Economic research on privacy regulation: Lessons from the gdpr and beyond. 2022.

Garrett A Johnson, Scott K Shriver, and Samuel G Goldberg. Privacy and market concentration: intended and unintended consequences of the gdpr. *Management Science*, 2023.

Justin P Johnson and David P Myatt. On the simple economics of advertising, marketing, and product design. *American Economic Review*, 96(3):756–784, 2006.

Justin P Johnson, Thomas Jungbauer, and Marcel Preuss. Online advertising, data sharing, and consumer control. *Management Science*, 2024.

Yuichiro Kamada and Fuhito Kojima. Voter preferences, polarization, and electoral policies. *American Economic Journal: Microeconomics*, 6(4):203–236, 2014.

T Tony Ke and K Sudhir. Privacy rights and data security: Gdpr and personal data markets. *Management Science*, 69(8):4389–4412, 2023.

T Tony Ke, Jiwoong Shin, and Jungju Yu. A model of product portfolio design: Guiding consumer search through brand positioning. *Marketing Science*, 42(6):1101–1124, 2023.

Eunji Kim, Yphtach Lelkes, and Joshua McCrain. Measuring dynamic media bias. *Proceedings of the National Academy of Sciences*, 119(32):e2202197119, 2022.

Dmitri Kuksov, Ron Shachar, and Kangkang Wang. Advertising and consumers' communications. *Marketing Science*, 32(2):294–309, 2013.

Dominique Olié Lauga, Elie Ofek, and Zsolt Katona. When and how should firms differentiate? quality and advertising decisions in a duopoly. *Journal of Marketing Research*, 59(6):1252–1265, 2022.

Estelle Laziuk. iOS 14.5 Opt-in Rate - Daily Updates Since Launch, 2021. URL `https://www.flurry.com/blog/ios-14-5-opt-in-rate-att-restricted-app-tracking-transparency-worldwide-us-daily-latest-update/`.

Yanzhe Lei, Sentao Miao, and Ruslan Momot. Privacy-preserving personalized revenue management. *Management Science*, 2023.

Jonathan Levin and Paul Milgrom. Online advertising: Heterogeneity and conflation in market design. *American Economic Review*, 100(2):603–607, 2010.

Song Lin. Two-sided price discrimination by media platforms. *Marketing Science*, 39(2):317–338, 2020.

Dina Mayzlin and Jiwoong Shin. Uninformative advertising as an invitation to search. *Marketing science*, 30(4):666–685, 2011.

Colleen McClain, Michelle Faverio, Monica Anderson, and Eugenie Park. How Americans View Data Privacy, 2023. URL `https://www.pewresearch.org/internet/2023/10/18/how-americans-view-data-privacy/`.

Marc J Melitz. The impact of trade on intra-industry reallocations and aggregate industry productivity. *econometrica*, 71(6):1695–1725, 2003.

Jeanine Miklós-Thal, Avi Goldfarb, Avery M Haviv, and Catherine Tucker. Digital hermits. Technical report, National Bureau of Economic Research, 2023.

Ruslan Momot and Marat Salikhov. Information elicitation from teams of privacy-conscious experts. *Available at SSRN 4248487*, 2022.

Sendhil Mullainathan and Andrei Shleifer. The market for news. *American economic review*, 95 (4):1031–1053, 2005.

Roger B Myerson. Optimal auction design. *Mathematics of operations research*, 6(1):58–73, 1981.

Z Eddie Ning, Jiwoong Shin, and Jungju Yu. Targeted advertising: Strategic mistargeting and personal data opt-out. *KAIST College of Business Working Paper Series*, 2023.

Novatiq. The evolution of programmatic advertising: State of the nation, 2024. URL `https://www.novatiq.com/the-evolution-of-programmatic-advertising-state-of-the-nation/`.

Eli Pariser. *The filter bubble: How the new personalized web is changing what we read and how we think*. Penguin, 2011.

Omid Rafieian and Hema Yoganarasimhan. Targeting and privacy in mobile advertising. *Marketing Science*, 40(2):193–218, 2021.

Andrew Rhodes and Jidong Zhou. Personalized pricing and privacy choice. Technical report, Working paper, 2021.

Manoel Horta Ribeiro, Raphael Ottoni, Robert West, Virgílio AF Almeida, and Wagner Meira Jr. Auditing radicalization pathways on youtube. In *Proceedings of the 2020 conference on fairness, accountability, and transparency*, pages 131–141, 2020.

Amin Sayedi. Real-time bidding in online display advertising. *Marketing Science*, 37(4):553–568, 2018.

Verena Schoenmueller, Oded Netzer, and Florian Stahl. Frontiers: Polarized america: From political polarization to preference polarization. *Marketing Science*, 42(1):48–60, 2023.

Jiwoong Shin and Woochoel Shin. A theory of irrelevant advertising: An agency-induced targeting inefficiency. *Management Science*, 69(8):4481–4497, 2023.

Jiwoong Shin and Jungju Yu. Targeted advertising and consumer inference. *Marketing Science*, 40 (5):900–922, 2021.

Philipp Strack and Kai Hao Yang. Privacy preserving signals. *Available at SSRN 4467608*, 2023.

Monic Sun and Feng Zhu. Ad revenue and content commercialization: Evidence from blogs. *Management Science*, 59(10):2314–2331, 2013.

Mike Sweeney. How Real-Time Bidding (RTB) Changed Online Display Advertising, 2023. URL `https://clearcode.cc/blog/real-time-bidding-online-display-advertising/`.

Curtis R Taylor. Consumer privacy and the market for customer information. *RAND Journal of Economics*, pages 631–650, 2004.

Catherine Tucker and Juanjuan Zhang. Growing two-sided networks by advertising the user base: A field experiment. *Marketing Science*, 29(5):805–814, 2010.

J Miguel Villas-Boas. Dynamic competition with customer recognition. *The Rand Journal of Economics*, pages 604–631, 1999.

J Miguel Villas-Boas. Price cycles in markets with customer recognition. *RAND Journal of Economics*, pages 486–501, 2004.

Kenneth C Wilbur. Advertising content and television advertising avoidance. *Journal of Media Economics*, 29(2):51–72, 2016.

Yi Xiang and Miklos Sarvary. News consumption and media bias. *Marketing Science*, 26(5): 611–628, 2007.

Jeremy Yang, Juanjuan Zhang, and Yuhan Zhang. Engagement that sells: Influencer video advertising on tiktok. *Marketing Science*, 2023.

Kai Hao Yang. Selling consumer data for profit: Optimal market-segmentation design and its consequences. *American Economic Review*, 112(4):1364–1393, 2022.

Yunfei Jesse Yao. Reputation for privacy. *Available at SSRN 4922734*, 2024.

Kaifu Zhang and Zsolt Katona. Contextual advertising. *Marketing Science*, 31(6):980–994, 2012.

Appendix

Proof of Lemma 1. Let p_j denote the posterior probability of $\theta = j$ conditional on the information $\mathcal{I}$, that is, $p_j = P(\theta = j \mid \mathcal{I})$. Now, we write the optimal ad choice as follows:

$$\begin{aligned}\max_a \mathbf{E}\left[\mathcal{M}(a;\theta) \mid \mathcal{I}\right] &= \max_a \left[1 - \gamma \mathbf{E}\left[(a-\theta)^2 \mid \mathcal{I}\right]\right] \\ &= \max_a \left[1 - \gamma\left(p_0 a^2 + p_{1/2}\left(a - \frac{1}{2}\right)^2 + p_1 (a-1)^2\right)\right]\end{aligned}$$

The first-order condition is $-\gamma\left(2p_0 a + 2p_{1/2}\left(a - \frac{1}{2}\right) + 2p_1(a-1)\right) = 0$, which results in $a^* = \frac{p_{1/2}+2p_1}{2} = \mathbf{E}[\theta \mid \mathcal{I}]$. ■

Proof of Corollary 1. The media firm's profit maximization problem is the following:

$$\begin{aligned}\max_{x,a} D(x) \cdot \mathbf{E}\left[\mathcal{M}(a;\theta) \mid \mathcal{I}\right] &= \max_x D(x) \cdot \mathbf{E}\left[\mathcal{M}(a^*;\theta) \mid \mathcal{I}\right] \\ &= \max_x D(x) \cdot \mathbf{E}\left[1 - \gamma(a^* - \theta)^2 \mid \mathcal{I}\right] \\ &= \max_x D(x) \cdot \left(1 - \gamma \mathbf{E}\left[(\mathbf{E}[\theta \mid \mathcal{I}] - \theta)^2 \mid \mathcal{I}\right]\right) \\ &= \max_x D(x) \cdot \left[1 - \gamma \mathbf{Var}(\theta|\mathcal{I})\right]\end{aligned}$$

■

Proof of Lemma 2. One can see that consumers whose type matches exactly the media firm's location will consume for sure. So, we just need to determine whether other consumers prefer consuming the content to the outside option.

1. Mainstream content strategy $x = 1/2$: The demand from mainstream consumers is $1 - 2\lambda$. Consider a type 0 consumer. Her utility from consuming the content is $U(1/2; 0) = \epsilon$. She will consume the content if and only if her consumption utility is greater than that of the outside option, i.e., $\epsilon > 0$. Therefore, the consumption probability is $P(\epsilon > 0) = 1 - F(0)$. Hence, the demand from type 0 consumers is $\lambda[1 - F(0)]$. By symmetry, the demand from type 1 consumers is also $\lambda[1-F(0)]$. In sum, the total demand is $1-2\lambda+2\lambda[1-F(0)] = 1-2F(0)\lambda$.

2. Niche content strategy $x = 0$: The demand from type 0 consumers is λ and the demand from type 1 consumers is 0. Consider a type 1/2 consumer. Her utility from consuming the content is $U(0, 1/2) = \epsilon$. She will consume the content if and only if $\epsilon > 0$, her utility from the outside option. By the same argument in the previous case, one can see that the demand from type 1/2 consumers is $(1 - 2\lambda)[1 - F(0)]$. In sum, the total demand is $\lambda + (1 - 2\lambda)[1 - F(0)] = 1 - F(0) + [2F(0) - 1]\lambda$.

3. Niche content strategy $x = 1$: It is symmetric to the previous $x = 0$ case.

■

Proof of Proposition 1. Under behavioral ad targeting, the match between the ad and the consumer type is always 1. Therefore, the media firm chooses the content strategy that leads to the highest demand. According to Lemma 2, a mainstream positioning generates a higher demand than niche content does if and only if $1 - 2F(0)\lambda > 1 - F(0) + [2F(0) - 1]\lambda \Leftrightarrow [4F(0) - 1]\lambda > F(0)$. ■

Proof of Lemma 3. Due to the symmetry between the two types of niche content, we only need to compare Privacy($\{0\}$) and Privacy($\{1/2\}$).

1. Mainstream content $x = 1/2$. According to Lemma 2, the demand from type 0 and from type 1 consumers are identical. So, $\mathbf{E}[\theta|x = 1/2] = 1/2$.

$$Privacy(\{1/2\}) = \mathbf{E}[(\theta - 1/2)^2|x = 1/2] = \frac{1}{2} \cdot \frac{\lambda[1 - F(0)]}{1 - 2\lambda F(0)}$$

2. Niche content $x = 0$.

$$\begin{aligned} Privacy(\{0\}) =& \mathbf{E}[\theta^2|x = 0] - \mathbf{E}[\theta|x = 0]^2 \\ =& \frac{[1 - F(0)](1 - 2\lambda)}{\lambda + [1 - F(0)](1 - 2\lambda)} \cdot (\frac{1}{2})^2 - \left\{ \frac{[1 - F(0)](1 - 2\lambda)}{1 - F(0) + [2F(0) - 1]\lambda} \cdot \frac{1}{2} \right\}^2 \\ =& \frac{1}{4} \cdot \frac{[1 - F(0)](1 - 2\lambda)\lambda}{\{\lambda + [1 - F(0)](1 - 2\lambda)\}^2} \end{aligned}$$

Therefore, $Privacy(\{0\}) < Privacy(\{1/2\})$

$$\Leftrightarrow \frac{1}{4} \cdot \frac{[1-F(0)](1-2\lambda)\lambda}{\{\lambda + [1-F(0)](1-2\lambda)\}^2} < \frac{1}{2} \cdot \frac{\lambda[1-F(0)]}{1-2\lambda F(0)}$$

$$\Leftrightarrow \lambda + \lambda(1-2\lambda)F(0) + 2\lambda(1-2\lambda)[1-F(0)] + \lambda^2 + [1-F(0)]^2(1-2\lambda)^2 > \frac{1}{2}$$

The LHS is greater than $\lambda+\lambda(1-2\lambda)F(0)+\lambda(1-2\lambda)[1-F(0)]+\lambda^2 = \lambda(2-\lambda)$. Since $\lambda \in (1/3, 1/2)$ and $\lambda(2-\lambda)$ increases in λ for $\lambda < 1$, we have the LHS $> \lambda(2-\lambda) > 1/3(2-1/3) = 5/9 > 1/2$. Therefore, $Privacy(\{0\}) < Privacy(\{1/2\})$ always holds. By definition, niche content is privacy-reducing over mainstream content. ■

Proof of Proposition 2. Consider contextual ad targeting. By symmetry, we only need to compare the firm's expected profits from type-0 niche content strategy and mainstream content strategy.

$$\begin{aligned}\pi^c(0) > \pi^c(1/2) \Leftrightarrow\ & D(0) \cdot [1 - \gamma \mathbf{Var}(\theta|x=0)] > D(1/2) \cdot [1 - \gamma \mathbf{Var}(\theta|x=1/2)] \\ \Leftrightarrow\ & \gamma > \frac{D(1/2)/D(0) - 1}{\mathbf{Var}(\theta|x=1/2)D(1/2)/D(0) - \mathbf{Var}(\theta|x=0)} =: \gamma^m.\end{aligned}$$

Since $D(1/2) > D(0)$ and $\mathbf{Var}[\theta|x=0] < \mathbf{Var}[\theta|x=1/2]$, we have $\gamma^m > 0$. So, the media firm chooses niche content strategy if $\gamma > \gamma^m$ and mainstream content strategy if $\gamma < \gamma^m$ under contextual ad targeting. Lastly, we show that γ^m, which can be expressed as $4\{F(0) + [1 - 4F(0)]\lambda\}\{1 - F(0) + [2F(0) - 1]\lambda\}/\{[1 - F(0)]\lambda[1 - 2F(0) + 4F(0)\lambda]\}$, decreases in λ.

(1) If $F(0) < 1/4$, then $4\{1 - F(0) + [2F(0) - 1]\lambda\}$ in the numerator decreases in λ while $[1 - F(0)][1 - 2F(0) + 4F(0)\lambda]$ in the denominator increases in λ. One can also see that the remaining part of $\gamma^m, \{F(0) + [1 - 4F(0)]\lambda\}/\lambda = F(0)/\lambda + 1 - 4F(0)$, decreases in λ.

(2) If $1/4 \leq F(0) \leq 1/2$, then both $F(0) + [1 - 4F(0)]\lambda$ and $1 - F(0) + [2F(0) - 1]\lambda$ decreases in λ. So, the numerator of γ^m decreases in λ. One can see that the denominator of γ^m increases in λ.

(3) If $F(0) > 1/2$, then $4\{F(0)+[1-4F(0)]\lambda\}$ in the numerator decreases in λ while $[1-F(0)][1-2F(0)+4F(0)\lambda]$ in the denominator increases in λ. One can also see that the remaining part of $\gamma^m, \{1 - F(0) + [2F(0) - 1]\lambda\}/\lambda = [1 - F(0)]/\lambda + 2F(0) - 1$, decreases in λ.

■

Proof of Proposition 3. We first compute the consumer welfare for each content strategy.

1. Mainstream content strategy $x = 1/2$: The consumer welfare is $(1 - 2\lambda)[1/2 + \mathbf{E}[\epsilon]] + 2[1 - F(0)]\lambda\mathbf{E}[\epsilon|\epsilon > 0] = 1/2 - \lambda + 2[1 - F(0)]\lambda\mathbf{E}[\epsilon|\epsilon > 0]$.

2. Niche content strategy $x = 0$: The consumer welfare is $\lambda[1/2+\mathbf{E}[\epsilon]]+[1-F(0)](1-2\lambda)\mathbf{E}[\epsilon|\epsilon > 0] = \lambda/2 + [1 - F(0)](1 - 2\lambda)\mathbf{E}[\epsilon|\epsilon > 0]$.

3. Niche content strategy $x = 1$: It is symmetric to the previous $x = 0$ case.

Niche content strategy leads to lower consumer welfare if and only if

$$\begin{aligned} &\lambda/2 + [1 - F(0)](1 - 2\lambda)\mathbf{E}[\epsilon|\epsilon > 0] < 1/2 - \lambda + 2[1 - F(0)]\lambda\mathbf{E}[\epsilon|\epsilon > 0] \\ \Leftrightarrow &\lambda < \hat{\lambda} = \frac{1 - 2[1 - F(0)]\mathbf{E}[\epsilon|\epsilon > 0]}{3 - 8[1 - F(0)]\mathbf{E}[\epsilon|\epsilon > 0]} \end{aligned}$$

In addition, $\hat{\lambda} > 1/3 \Leftrightarrow 2[1 - F(0)]\mathbf{E}[\epsilon|\epsilon > 0] > 0$, which holds for any distribution of ϵ. ■

Proof of Proposition 4. We first simplify the demand in Lemma 4 when $\epsilon \sim U[-1/2, 1/2]$.

Corollary 4. *Suppose $\epsilon \sim U[-1/2, 1/2]$. The demand under each possible equilibrium is:*

1. *Under (0,1) strategy profile, Firm 1's total demand is $3/8 + \lambda/4$. Among them, λ are type 0 consumers and $3/8 - 3\lambda/4$ are mainstream consumers. Firm 2's total demand is $3/8 + \lambda/4$. Among them, λ are type 1 consumers and $3/8 - 3\lambda/4$ are mainstream consumers.*

2. *Under (1/2,1/2) strategy profile, each firm's total demand is $1/2 - \lambda/4$. Among them, $1/2 - \lambda$ are mainstream consumers, $3\lambda/8$ are type 0 consumers, and $3\lambda/8$ are type 1 consumers.*

3. *Under (0,1/2) strategy profile, Firm 1's total demand is $1/8 + 5\lambda/8$. Among them, $7\lambda/8$ are type 0 consumers and $(1 - 2\lambda)/8$ are mainstream consumers. Firm 2's total demand is $7/8 - 9\lambda/8$. Among them, $\lambda/8$ are type 0 consumers, $\frac{7}{8}(1 - 2\lambda)$ are type 1/2 consumers, and $\lambda/2$ are type 1 consumers.*

Now consider the equilibrium strategy under behavioral ad targeting. Under behavioral ad targeting, the advertising revenue per consumer is 1. So, each firm's profit equals its total demand. Suppose the equilibrium is (0,1/2). Firm 1 will deviate from $x = 0$ to $x = 1/2$ if and only if:

$$1/2 - \lambda/4 > 1/8 + 5\lambda/8 \Leftrightarrow \lambda < \lambda_2 = 3/7 \tag{6}$$

Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:

$$3/8 + \lambda/4 > 7/8 - 9\lambda/8 \Leftrightarrow \lambda > \lambda_1 = 4/11 \tag{7}$$

From the above conditions and noting that $\lambda_1 < \lambda_2$, one can see that (0,1/2) will never be an equilibrium.

Now suppose the equilibrium is (1/2,1/2). Equation (6) implies that the firm will deviate to 0 or 1 if $\lambda > \lambda_2$. Similarly, if the equilibrium is (0,1), Equation (7) implies that the firm will deviate to 1/2 if $\lambda < \lambda_1$.

In sum, the equilibrium is (1/2,1/2) if $\lambda < \lambda_1$, (1/2,1/2) or (0,1) if $\lambda_1 < \lambda < \lambda_2$, and (0,1) if $\lambda > \lambda_2$. ■

Proof of Proposition 5. We first characterize the equilibrium ad choices and profits for each firm under contextual ad targeting.

1. Under (1/2,1/2) strategy profile, by symmetry, $a = \mathbf{E}(\theta) = 1/2$. $\mathbf{Var}(\theta) = \mathbf{E}[\theta - \mathbf{E}(\theta)]^2 = 3\lambda/[4(2-\lambda)]$.

$$\pi(1/2, 1/2) = D(1/2, 1/2) \cdot [1 - \gamma \mathbf{Var}(\theta)] = \frac{2-\lambda}{4} - \frac{3\gamma\lambda}{16}$$

2. Under (0,1) strategy profile, by symmetry, we only need to consider firm 1. Firm 2's ad choice will be $1 - a_1$, and firm 2's profit will be identical to firm 1's. We have $a_1 = \mathbf{E}(\theta) = 3(1-2\lambda)/[2(3+2\lambda)], \mathbf{Var}(\theta) = \mathbf{E}[\theta - \mathbf{E}(\theta)]^2 = 6\lambda(1-2\lambda)/(3+2\lambda)^2$.

$$\pi(0, 1) = D(0, 1) \cdot [1 - \gamma \mathbf{Var}(\theta)] = \frac{2\lambda+3}{8} - \frac{3\gamma\lambda(1-2\lambda)}{4(2\lambda+3)}.$$

3. Under (0,1/2) strategy profile, consider firm 1 first. We have $a_1 = \mathbf{E}(\theta) = \mathbf{E}_1(\theta) = (1-2\lambda)/[2(1+5\lambda)], \mathbf{Var}_1(\theta) = \mathbf{E}_1[\theta - \mathbf{E}_1(\theta)]^2 = 7\lambda(1-2\lambda)/[4(1+5\lambda)^2]$.

$$\pi_1(0, 1/2) = D_1(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_1(\theta)] = \frac{1+5\lambda}{8} - \frac{7\gamma\lambda(1-2\lambda)}{32(1+5\lambda)}.$$

Consider firm 2 then. We have $a_2 = \mathbf{E}_2(\theta) = (7-6\lambda)/[2(7-9\lambda)], \mathbf{Var}_2(\theta) = \mathbf{E}_2[\theta - \mathbf{E}_2(\theta)]^2 = (35\lambda - 54\lambda^2)/[4(7-9\lambda)^2]$.

$$\pi_2(0, 1/2) = D_2(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_2(\theta)] = \frac{7-9\lambda}{8} - \frac{\gamma(35\lambda - 54\lambda^2)}{32(7-9\lambda)}.$$

Suppose the equilibrium is (0,1/2). Firm 1 will deviate from $x = 0$ to $x = 1/2$ if and only if:

$$\pi(1/2, 1/2) > \pi_1(0, 1/2) \Leftrightarrow \gamma < \gamma^d = (\lambda_2 - \lambda)\frac{28(1+5\lambda)}{\lambda(44\lambda - 1)} \tag{8}$$

An immediate implication is that this deviation never happens if $\lambda \geq \lambda_2$.

Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:[23]

$$\pi(0, 1) > \pi_2(0, 1/2) \Leftrightarrow \gamma > \gamma^{d'} = (\lambda_1 - \lambda)\frac{44(2\lambda+3)(7-9\lambda)}{\lambda(-540\lambda^2 + 460\lambda - 63)} \tag{9}$$

An immediate implication is that deviation always happens if $\lambda \geq \lambda_1$.[24] Therefore, (0,1/2) may only be an equilibrium if $\lambda < \lambda_1$. Now suppose $\lambda < \lambda_1$. In this case, we have shown that firm 1

[23] Firm 2 has a stronger incentive to deviate to $x = 1$ rather than $x = 0$ to soften competition. So, we only need to consider its deviation to $x = 1$.

[24] One can check that $44(2\lambda+3)(7-9\lambda)/[\lambda(-540\lambda^2 + 460\lambda - 63)] > 0$ for $\lambda \in (1/3, 1/2)$.

will deviate if $\gamma < \gamma^d$ and firm 2 will deviate if $\gamma > \gamma^{d'}$. Some calculation yields that there exists a unique $\lambda_0 \in (1/3, \lambda_1)$ such that $\gamma^d < (>)\gamma^{d'}$ if $\lambda < (>)\lambda_0$. Therefore, at least one deviation happens and (0,1/2) will not be an equilibrium if $\lambda > \lambda_0$. There is no deviation and (0,1/2) is an equilibrium if $\lambda < \lambda_0$ and $\gamma^d < \gamma < \gamma^{d'}$.

Now suppose the equilibrium is (1/2,1/2). Equation (8) implies that the firm will deviate to 0 or 1 if $\gamma > \gamma^d$, which may hold if $\lambda < \lambda_2$ and always holds if $\lambda > \lambda_2$. Similarly, if the equilibrium is (0,1), Equation (9) implies that the firm will deviate to 1/2 if $\gamma < \gamma^{d'}$, which may hold if $\lambda < \lambda_1$ and never holds if $\lambda > \lambda_1$. Therefore, (0,1) is an equilibrium if $\lambda > \lambda_1$ or if $\lambda < \lambda_1$ & $\gamma > \gamma^{d'}$. (1/2,1/2) is an equilibrium if $\lambda < \lambda_2$ & $\gamma < \gamma^d$.

We summarize the equilibria in Table 1. ■

Table 1: Complete equilibria under behavioral and contextual ad targeting in duopoly.

Condition	Equilibrium Under	
	Behavioral Ad Targeting	Contextual Ad Targeting
$1/3 < \lambda < \lambda_1$ & $\gamma < \min\{\gamma^d, \gamma^{d'}\}$	(1/2,1/2)	(1/2,1/2)
$1/3 < \lambda < \lambda_0$ & $\gamma^d < \gamma < \gamma^{d'}$	(1/2,1/2)	(0,1/2)
$\lambda_0 < \lambda < \lambda_1$ & $\gamma^{d'} < \gamma < \gamma^d$	(1/2,1/2)	(1/2,1/2) or (0,1)
$1/3 < \lambda < \lambda_1$ & $\gamma > \max\{\gamma^d, \gamma^{d'}\}$	(1/2,1/2)	(0,1)
$\lambda_0 < \lambda_1 < \lambda < \lambda_2$ & $\gamma < \gamma^d$	(1/2,1/2) or (0,1)	(1/2,1/2) or (0,1)
$\lambda_1 < \lambda < \lambda_2$ & $\gamma > \gamma^d$	(1/2,1/2) or (0,1)	(0,1)
$\lambda > \lambda_2$	(0,1)	(0,1)

Online Appendix for Privacy and Polarization: An Inference-Based Framework

1 Ad Revenue Under a Competitive Auction Environment

In the main text, we consider a case where the media firm chooses each ad and extracts the match value as revenue. In reality, however, the media firm does not often have this level of control over ad allocation because advertising slots are sold through auctions where advertisers compete for impressions. These auctions are generally sold by ad intermediaries and platforms such as Google Ads, and the intermediary collects an α proportion (e.g., 30%) of the total revenue extracted from advertisers. In this section, we show that if the media firm delegates this decision to an auctioneer and allows ads to compete in an auction environment, the model setup remains unchanged.

Auction Environment: Suppose there is a large number of infinitesimal ad campaigns located on $[0, 1]$ who compete in an auction environment. Following the convention of this literature, the auctioneer runs a standard auction, such as a second- or first-price auction. We focus on the second-price auction in our analysis here, but one can show the revenue equivalence for the case of first-price (Myerson, 1981).

Advertisers' Bidding Behavior: Each ad a receives value $1 - \gamma(a - \theta)^2$ from being shown to a type-θ consumer. However, advertisers do not necessarily know the consumer's type and need to form beliefs about it given the information available. As such, their ex-ante value of an impression given information $\mathcal{I}$ about that impression is $\mathbf{E}[1 - \gamma(a - \theta)^2 \mid \mathcal{I}]$. Under behavioral targeting, this ex-ante valuation is $1 - \gamma(a - \theta)^2$ because the advertiser knows the consumer's type θ. However, under contextual targeting, the ex-ante valuation for an impression is $1-\gamma\mathbf{E}[(a-\theta)^2 \mid x]$ because the information available is only the consumer's content choice x. Because advertisers are competing in a second-price auction where truth-telling is the equilibrium bidding behavior, each ad a submits the bid $1 - \gamma\mathbf{E}[(a - \theta)^2 \mid \mathcal{I}]$ depending on the information available.

Expected Ad Revenue: In a second-price auction with infinitesimal ad campaigns, the impression is allocated to the ad with the highest bid, that is, $a^* = \arg\max_a 1 - \gamma\mathbf{E}[(a - \theta)^2 \mid \mathcal{I}]$. One could easily use the main property of variance and show that $a^* = \mathbf{E}[\theta \mid \mathcal{I}]$ (a formal proof is provided in Lemma 1). Because there are multiple bidders at a^*, the auctioneer can extract all the value as the total ad revenue for the impression with information $\mathcal{I}$ under the second-price auction. We define the firm's ad revenue for an impression with information $\mathcal{I}$ as $\text{AdRev}(\mathcal{I})$ as follows:

$$\begin{aligned}\text{AdRev}(\mathcal{I}) &= (1 - \alpha)\left(1 - \gamma\mathbf{E}\left[(\mathbf{E}[\theta \mid x] - \theta)^2 \mid \mathcal{I}\right]\right) \\ &= (1 - \alpha)\left(1 - \gamma\mathbf{Var}(\theta \mid \mathcal{I})\right),\end{aligned}$$

where $(1 - \alpha)$ is the share of total revenue that is collected by the media firm.

Media Firm's Profit Maximization: Based on the ad revenue collected through auctions in a setting where the media firm does not have full control over ad allocation, the firm's profit maximization problem can be written as follows:

$$\max_x D(x)\text{AdRev}(\mathcal{I})$$

It is easy to show that this maximization problem is equivalent to the one we use in the main text as shown in Corollary 1. Therefore, the media firm's full control over ad allocation is not a requirement for our main results and we will arrive at the same insights if advertisers can self-select into impressions in an auction environment.

In light of this equivalence, one could view the value of inference from the point-of-view of advertisers in a market environment. If consumers' self-selection into content provides sharper inference about the consumer type, this information will be reflected in advertisers' bids, which translates into higher revenues for the media firm.

2 Proofs

Proof of Lemma 4. There are three candidate equilibria: (0,1/2), (1/2,1/2), (0,1).

1. Under (0,1) strategy profile, the demands for type 0 and type 1 content are symmetric. So, we only need to examine type 0 content. Each consumer will choose type 0 content if and only if her utility from type 0 content is positive (higher than the outside option) and higher than her utility from type 1 content.

   One can see that all type 0 consumers will choose type 0 content. Now consider type 1/2 consumers. A mainstream consumer will consume one of the contents if and only if her utility from at least one content is positive. By symmetry, her overall probabilities of choosing type 0 and type 1 content are identical.

$$\begin{aligned} & P(\text{a mainstream consumer chooses type 0 content}) \\ = & \frac{1}{2} \cdot P(\max\{\epsilon_0, \epsilon_1\} > 0) \\ \overset{\textit{independence of } \epsilon_j}{=} & \frac{1}{2} \cdot [1 - P(\epsilon_0 \leq 0)P(\epsilon_1 \leq 0)] \\ = & \frac{1}{2} \cdot [1 - F(0)^2] \end{aligned}$$

   In sum, the demand of type 0 content from type 0 consumers is λ and from type 1/2 consumers is $[1 - F(0)^2](1 - 2\lambda)/2$. The total demand of type 0 content is $\lambda + [1 - F(0)^2](1 - 2\lambda)/2$.

2. Under (1/2, 1/2) strategy profile, one can see that all mainstream consumers will consume the content by one of the media firms. The demands from type 0 and type 1 consumers are

symmetric. Consider type 0 consumers. She will consume the content by one of the firms if and only if her utility from at least one content is positive. By symmetry, her overall probabilities of choosing either mainstream content are identical.

$$\begin{aligned} & P(\text{a type 0 consumer chooses firm } i) \\ = & \frac{1}{2} \cdot P(\max\{\epsilon_{1/2}, \epsilon_{1/2'}\} > 0) \\ \overset{\textit{independence of } \epsilon_j}{=} & \frac{1}{2} \cdot [1 - P(\epsilon_{1/2} \leq 0) P(\epsilon_{1/2'} \leq 0)] \\ = & \frac{1 - F(0)^2}{2} \end{aligned}$$

In sum, the demand of either firm from mainstream consumers is $(1-2\lambda)/2$, from type 0 consumers is $[1-F(0)^2]\lambda/2$, and from type 1 consumers is $\frac{1-F(0)^2}{2}\lambda$. The total demand of either firm is $(1-2\lambda)/2 + [1-F(0)^2]\lambda/2$.

3. Under (0, 1/2) strategy profile, consider first the demand for type 0 content. Since a type 0 consumer's utility from consuming type 0 content is always positive, we only need to compare her utility from type 0 and type 1/2 contents.

$$\begin{aligned} & P(\text{a type 0 consumer chooses type 0 content}) \\ = & P(1/2 + \epsilon_0 > 0 + \epsilon_{1/2}) \\ = & P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\ = & 1 - F(0) + \int_{-1/2}^{0} \int_{-1/2}^{\epsilon_0 + 1/2} f(\epsilon_{1/2}) d\epsilon_{1/2} f(\epsilon_0) d\epsilon_0 \\ = & 1 - F(0) + \int_{-1/2}^{0} F(\epsilon_0 + 1/2) f(\epsilon_0) d\epsilon_0 \\ \overset{\text{integral by parts}}{=} & 1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0 \end{aligned}$$

A type 1/2 consumer will choose type 0 content if and only if her utility of consuming it is positive and higher than her utility of consuming mainstream content. Since her utility of consuming mainstream content is always positive, we only need the condition that her utility of consuming type 0 content is higher than her utility of consuming mainstream content.

$$\begin{aligned} & P(\text{a type 1/2 consumer chooses type 0 content}) \\ = & P(0 + \epsilon_0 > 1/2 + \epsilon_{1/2}) \\ = & P(\epsilon_0 > \epsilon_{1/2} + 1/2) \\ \overset{\textit{symmetry}}{=} & P(\epsilon_{1/2} > \epsilon_0 + 1/2) \end{aligned}$$

$$
\begin{aligned}
&= 1 - P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&\overset{previous\ case}{=} 1 - [1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] \\
&= \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

Therefore, the demand of type 0 content from type 0 consumers is $\lambda[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0]$ and from type 1/2 consumers is $(1-2\lambda) \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0+1/2) d\epsilon_0$. The total demand of of type 0 content is $\lambda[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] + (1 - 2\lambda) \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$. Consider now the demand for type 1/2 content.

$$
\begin{aligned}
&P(\text{a type 0 consumer chooses type 1/2 content}) \\
&= 1 - P(\text{a type 0 consumer chooses type 0 content}) \\
&= \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

$$
\begin{aligned}
&P(\text{a type 1/2 consumer chooses type 1/2 content}) \\
&= 1 - P(\text{a type 1/2 consumer chooses type 0 content}) \\
&= 1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

$$
\begin{aligned}
&P(\text{a type 1 consumer chooses type 1/2 content}) \\
&= P(U(1/2, 1) > 0) \\
&= P(\epsilon_{1/2} > 0) \\
&= 1 - F(0)
\end{aligned}
$$

Therefore, the demand of type 1/2 content from type 0 consumers is $\lambda \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$, from type 1/2 consumers is $(1 - 2\lambda)[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0]$, and from type 1 consumers is $[1 - F(0)]\lambda$. The total demand of type 1/2 content is $\lambda \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0 + (1 - 2\lambda)[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] + [1 - F(0)]\lambda$.

■

Proof of Proposition 6. We first compute the consumer utility from content consumption for each possible equilibrium strategy profile.

1. Under (0,1) strategy profile, the consumer utility from consuming each firm's content is:

$$\begin{aligned}&\lambda(1/2+\mathbf{E}[\epsilon_0])+(3/8-3\lambda/4)\mathbf{E}[\epsilon_0|\epsilon_0>0 \text{ and } \epsilon_0>\epsilon_1]\\ =&\lambda/2+(3/8-3\lambda/4)\cdot 5/18=\frac{5-\lambda}{18}\end{aligned}$$

   Hence, the total consumer utility from content consumption is $2(5-\lambda)/18=(5-\lambda)/9$.

2. Under (1/2,1/2) strategy profile, the consumer utility from consuming each firm's content is:

$$\begin{aligned}&\frac{1-2\lambda}{2}(1/2+\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>\epsilon_{1/2'}])+2(\frac{1-F(0)^2}{2})\lambda\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0 \text{ and } \epsilon_{1/2}>\epsilon_{1/2'}]\\ =&\frac{1-2\lambda}{3}+\frac{3\lambda}{4}\frac{5}{18}=\frac{8-11\lambda}{24}\end{aligned}$$

   Hence, the total consumer utility from content consumption is $2(6-7\lambda)/24=(6-7\lambda)/12$.

3. Under (0,1/2) strategy profile, the consumer utility from consuming firm 1's content is:

$$\begin{aligned}&\frac{7\lambda}{8}(1/2+\mathbf{E}[\epsilon_0|\epsilon_0+1/2>\epsilon_{1/2}])+\frac{1-2\lambda}{8}\mathbf{E}[\epsilon_0|\epsilon_0>0 \text{ and } \epsilon_0>1/2+\epsilon_{1/2}]\\ =&\frac{7\lambda}{8}\cdot(\frac{1}{2}+\frac{1}{21})+\frac{1-2\lambda}{8}\cdot\frac{1}{3}=\frac{2+19\lambda}{48}\end{aligned}$$

   The consumer utility from consuming firm 2's content is:

$$\begin{aligned}&\frac{\lambda}{8}\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0 \text{ and } \epsilon_{1/2}>1/2+\epsilon_0]+\frac{7(1-2\lambda)}{8}(1/2+\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>1/2+\epsilon_0)+\\ &\frac{\lambda}{2}\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0]\\ =&\frac{\lambda}{8}\cdot\frac{1}{3}+\frac{7(1-2\lambda)}{8}(1/2+1/3)+\frac{\lambda}{2}\cdot\frac{1}{4}=\frac{35-62\lambda}{48}\end{aligned}$$

   Hence, the total consumer utility from content consumption is $(2+19\lambda)/48+(35-62\lambda)/48=(37-43\lambda)/48$.

The total consumer utility from content consumption under (0,1) strategy profile is higher than that under (0,1/2) strategy profile if and only if $2\cdot(5-\lambda)/18>(37-43\lambda)/48\Leftrightarrow\lambda>31/113$, which always holds. The total consumer utility from content consumption under (0,1/2) strategy profile is higher than that under (1/2,1/2) strategy profile if and only if $(37-43\lambda)/48>2(8-11\lambda)/24\Leftrightarrow\lambda>-5$, which always holds. ■

Proof of Proposition 7. We consider two cases:

1. The base utility $v<1/2$: In this case, the consumer may not consume the content even if the content location perfectly match the consumer type. In addition, the consumer never consumes the content if the content location is on the opposite end of the consumer's location.

(a) Monopoly

We first summarize the demand for a monopoly by choosing a particular content.

Suppose the monopoly chooses niche content strategy $x = 0$.[1] One can show that the demand from type 0 consumer is $\lambda(1/2 + v)$, from type 1/2 consumer is $(1 - 2\lambda)v$, and from type 1 consumer is 0. Hence, the total demand is $\lambda(1/2 + v) + (1 - 2\lambda)v$.

Suppose the monopoly chooses mainstream content strategy $x = 1/2$. One can show that the demand from type 0 consumer is λv, from type 1/2 consumer is $(1 - 2\lambda)(1/2 + v)$, and from type 1 consumer is λv. Hence, the total demand is $(1 - 2\lambda)(1/2 + v) + 2\lambda v$.

$$D(0) < D(1/2) \Leftrightarrow \lambda < \frac{1}{3 - 2v}$$

Consequently, the monopoly chooses mainstream positioning under behavioral ad targeting if and only if $\lambda < 1/(3 - 2v)$.

Now let us study the equilibrium under contextual ad targeting. We restrict the attention to the case where $1/3 < \lambda < 1/(3 - 2v)$ because niche positioning gives the firm higher demand and better inference if $\lambda \geq 1/(3-2v)$ whereas mainstream positioning dominates niche positioning if $\lambda \leq 1/3$.

Some calculations yield that $\mathbf{Var}(\theta|x = 0) = v(1-2\lambda)(\lambda v+\lambda/2)/[4(\lambda/2-\lambda v+v)^2]$ and $\mathbf{Var}(\theta|x = 1/2) = \lambda v/(1 + 2v - 2\lambda)$. Niche positioning gives the firm better inference if and only if $\mathbf{Var}(\theta|x = 0) < \mathbf{Var}(\theta|x = 1/2)$, which always hold for $\lambda \in (1/3, 1/(3-2v))$. The monopoly prefers niche position to mainstream position if:

$$\begin{aligned}
&\pi_{con}(x = 0) > \pi_{con}(x = 1/2) \\
\Leftrightarrow &D(0)[1 - \gamma\mathbf{Var}(\theta|x = 0) > D(1/2)[1 - \gamma\mathbf{Var}(\theta|x = 1/2)] \\
\Leftrightarrow &\gamma > \frac{16[1 + \lambda(2v - 3)]}{\lambda v[2\lambda^3(1 - 2v)^2(1 + 2v) + 4\lambda v(-1 + 2v + 8v^2) + \lambda^2(-1 + 10v + 4v^2 - 40v^3) - 4(-4 + v^2 + 2v^3)]}
\end{aligned}$$

Denote the threshold as γ^m. One can show that γ^m decreases in λ and increases in v.[2]

Now consider consumer utility from content consumption. Denote the consumer utility from content consumption given content strategy x as $CW(x)$. We have:

$$\begin{aligned}
CW(1/2) =&(1 - 2\lambda)\mathbf{E}[v + \epsilon|v + \epsilon > 0]P(v + \epsilon > 0)+ \\
&2\lambda\mathbf{E}[v - 1/2 + \epsilon|v - 1/2 + \epsilon > 0]P(v - 1/2 + \epsilon > 0) \\
=&\frac{(1 - 2\lambda)(v + 1/2)^2}{2} + \lambda v^2 \\
CW(0) =&\lambda\mathbf{E}[v + \epsilon|v + \epsilon > 0]P(v + \epsilon > 0)+ \\
&(1 - 2\lambda)\mathbf{E}[v - 1/2 + \epsilon|v - 1/2 + \epsilon > 0]P(v - 1/2 + \epsilon > 0)
\end{aligned}$$

[1] The case where $x = 1$ is symmetric to this case. We only need to consider one of these cases.

[2] Observe that γ^m can be negative for small v. In that case, we let $\gamma^m = 0$ for convenience.

$$=\frac{\lambda(v+1/2)^2}{2}+\frac{(1-2\lambda)v^2}{2}$$

Niche content strategy leads to lower consumer welfare if: $CW(0) < Cw(1/2) \Leftrightarrow \lambda < \hat{\lambda} := (v+1/4)/(-v^2+3v+1/4)$. One can see that $\hat{\lambda} > 1/3$.

(b) Duopoly

We first compute the demand given different strategy profiles.

i. Under (0,1) strategy profile, the demand for type 0 content:
From $\theta = 0$ consumers: $\lambda P(v+\epsilon > 0) = \lambda(v+1/2)$.
From $\theta = 1/2$ consumers: $(1/2)(1-2\lambda)P(\max\{v-1/2+\epsilon_0, v-1/2+\epsilon_1\} > 0) = (1-2\lambda)(2v-v^2)/2$.
From $\theta = 1$ consumers: 0.
Total demand: $\lambda(v+1/2)+(1-2\lambda)(2v-v^2)/2$.
The demand for type 1 content is symmetric.

ii. Under (1/2, 1/2) strategy profile, the demand for type 1/2 content:
From $\theta = 0/1$ consumers: $(1/2)\lambda P(\max\{v-1/2+\epsilon_{1/2}, v-1/2+\epsilon_{1/2'}\} > 0) = \lambda(2v-v^2)/2$.
From $\theta = 1/2$ consumers: $(1/2)(1-2\lambda)P(\max\{v+\epsilon_{1/2}, v+\epsilon_{1/2'}\} > 0) = (1-2\lambda)[1-(1/2-v)^2]/2$.
Total demand: $\lambda(2v-v^2)+(1-2\lambda)[1-(1/2-v)^2]/2$.

iii. Under (0,1/2) strategy profile, the demand for type 0 content:
From $\theta = 0$ consumers: $\lambda P(v+\epsilon_0 > 0 \textit{ and } v+\epsilon_0 > v-1/2+\epsilon_{1/2}) = \lambda(v/2+5/8)$.
From $\theta = 1/2$ consumers: $(1/2)\lambda P(v-1/2+\epsilon_0 > 0 \textit{ and } v-1/2+\epsilon_0 > v+\epsilon_{1/2}) = (1-2\lambda)(v-v^2)/2$.
From $\theta = 1$ consumers: 0.
Total demand: $\lambda(v/2+5/8)+(1-2\lambda)(v-v^2)/2$.

The demand for type 1/2 content:
From $\theta = 0$ consumers: $\lambda(v-v^2)/2$.
From $\theta = 1/2$ consumers: $(1-2\lambda)(v/2+5/8)$.
From $\theta = 1$ consumers: $\lambda P(v-1/2+\epsilon_{1/2} > 0) = \lambda v$.
Total demand: $\lambda(v-v^2)/2+(1-2\lambda)(v/2+5/8)+\lambda v$.

Consider the equilibrium under behavioral ad targeting. Suppose (0,1/2) is an equilibrium. Firm 1 will deviate from $x=0$ to $x=1/2$ if and only if:

$$\lambda(2v-v^2)+\frac{(1-2\lambda)[1-(1/2-v)^2]}{2} > \lambda(v/2+5/8)+\frac{(1-2\lambda)(v-v^2)}{2}$$
$$\Leftrightarrow \lambda < \lambda_2 := \frac{3}{11-12v+8v^2}(< \frac{1}{3-2v})$$

Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:

$$\lambda(v + 1/2) + \frac{(1-2\lambda)(2v - v^2)}{2} > \frac{\lambda(v - v^2)}{2} + (1-2\lambda)(v/2 + 5/8) + \lambda v$$
$$\Leftrightarrow \lambda > \lambda_1 := \frac{5 - 4v + 4v^2}{14 - 12v + 12v^2}$$

One can see that λ_1 and λ_2 increases in v. One can also show that there exists $\underline{v} < 7/25$ such that $\lambda_1 < \lambda_2$ when $\underline{v} < v < 1/2$. Therefore, (0,1/2) will never be an equilibrium if $\underline{v} < v < 1/2$. We assume that $\underline{v} < v < 1/2$ in subsequent analyses.
Now suppose the equilibrium is (1/2,1/2). One can see that the firm will deviate to 0 or 1 if $\lambda > \lambda_2$. Similarly, if the equilibrium is (0,1), the firm will deviate to 1/2 if $\lambda < \lambda_1$.
In sum, the equilibrium is (1/2,1/2) if $\lambda < \lambda_1$, (1/2,1/2) or (0,1) if $\lambda_1 < \lambda < \lambda_2$, and (0,1) if $\lambda > \lambda_2$.

Now consider the equilibrium under contextual ad targeting.
We first characterize the equilibrium ad choices and profits for each firm.

A. Under (1/2,1/2) strategy profile
By symmetry, $a = \mathbf{E}(\theta) = 1/2$. $\mathbf{Var}(\theta) = \lambda v(2-v)/[3 + 4v - 4v^2 + \lambda(4v^2 - 6)]$.

$$\begin{aligned}\pi(1/2, 1/2) =& D(1/2, 1/2) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\ =& \lambda(2v - v^2) + \frac{(1-2\lambda)[1 - (1/2 - v)^2]}{2} - \frac{\gamma\lambda(2v - v^2)}{4}\end{aligned}$$

B. Under (0,1) strategy profile
By symmetry, we only need to consider firm 1. Firm 2's ad choice will be $1 - a_1$, and firm 2's profit will be identical to firm 1's.
$\mathbf{Var}(\theta) = \frac{1}{4}\frac{(1-2\lambda)(2v-v^2)/2}{(v+1/2)\lambda+(1-2\lambda)(2v-v^2)/2}\frac{(v+1/2)\lambda}{(v+1/2)\lambda+(1-2\lambda)(2v-v^2)/2}$.

$$\begin{aligned}\pi(0, 1) =& D(0, 1) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\ =& (v + 1/2)\lambda + (1-2\lambda)(2v - v^2)/2 - \gamma\frac{1}{4}\frac{\frac{1}{2}(1-2\lambda)(2v - v^2)(v + 1/2)\lambda}{(v + 1/2)\lambda + (1-2\lambda)(2v - v^2)/2}\end{aligned}$$

C. Under (0,1/2) strategy profile
Consider firm 1 first.

$$\mathbf{Var}_1(\theta) = \frac{1}{4}\frac{(1-2\lambda)(v - v^2)/2}{(v/2 + 5/8)\lambda + (1-2\lambda)(v - v^2)/2}\frac{(v/2 + 5/8)\lambda}{(v/2 + 5/8)\lambda + (1-2\lambda)(v - v^2)/2}.$$

$$\pi_1(0, 1/2) = D_1(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_1(\theta)]$$

$$=(\frac{v}{2}+\frac{5}{8})\lambda+\frac{(1-2\lambda)(v-v^2)}{2}-\gamma\frac{1}{4}\frac{\frac{1}{2}(1-2\lambda)(v-v^2)(v/2+5/8)\lambda}{(v/2+5/8)\lambda+(1-2\lambda)(v-v^2)/2}$$

Consider firm 2 then.

$$\mathbf{Var}_2(\theta)=\frac{(1-2\lambda(v/2+5/8)}{D_2(0,1/2)}\frac{1}{4}+\frac{\lambda v}{D_2(0,1/2)}\cdot 1-[\frac{(1-2\lambda(v/2+5/8)}{D_2(0,1/2)}\frac{1}{2}+\frac{\lambda v}{D_2(0,1/2)}\cdot 1]^2.$$

$$\begin{aligned}\pi_2(0,1/2)=&D_2(0,1/2)\cdot[1-\gamma\mathbf{Var}_2(\theta)]\\=&\frac{1}{8}[5+4v-2\lambda(5-2v+2v^2)][1+\frac{\gamma\lambda v[-15-7v+4v^2+6\lambda(5-3v+4v^2)]}{[5+4v-2\lambda(5-2v+2v^2)]^2}].\end{aligned}$$

Suppose the equilibrium is (0,1/2). One can show that there exists a γ^d such that firm 1 will deviate from $x=0$ to $x=1/2$ if and only if:

$$\pi(1/2,1/2)>\pi_1(0,1/2)\Leftrightarrow\gamma<\gamma^d$$

Moreover, γ^d increases in v for $v\in(0,1/2)$ and $\lambda\in(1/3,1/2)$. Given that $1/3<\lambda<1/2$ and $0<v<1/2$, we have $\gamma^d>0\Leftrightarrow(3-\sqrt{5})/4<v<1/2$ and $1/3<\lambda<\lambda_2$. An immediate implication is that this deviation never happens if $\lambda\geq\lambda_2$.
One can show that there exists a $\gamma^{d'}$ such that firm 2 will deviate from $x=1/2$ to $x=1$ if and only if:[3]

$$\pi(0,1)>\pi_2(0,1/2)\Leftrightarrow\gamma>\gamma^{d'}$$

Moreover, given that $1/3<\lambda<1/2$ and $0<v<1/2$, we have $\gamma^{d'}\leq 0\Leftrightarrow\lambda_1\leq\lambda<1/2$. An immediate implication is that deviation always happens if $\lambda\geq\lambda_1$. So, (0,1/2) may only be an equilibrium if $\lambda<\lambda_1$. Now suppose $\lambda<\lambda_1$. In this case, we have shown that firm 1 will deviate if $\gamma<\gamma^d$ and firm 2 will deviate if $\gamma>\gamma^{d'}$.
Now suppose the equilibrium is (1/2,1/2). One can see that the firm will deviate to 0 or 1 if $\gamma>\gamma^d$, which may hold if $\lambda<\lambda_2$ and always holds if $\lambda>\lambda_2$. Similarly, if the equilibrium is (0,1), the firm will deviate to 1/2 if $\gamma<\gamma^{d'}$, which may hold if $\lambda<\lambda_1$ and never holds if $\lambda>\lambda_1$.
Since $\gamma^m>\max\{\gamma^d,\gamma^{d'}\}$, $\forall v\in(\underline{v},1/2),\lambda\in(1/3,1/2)$, Let us define $\gamma^{d''}$ by $\min\{\gamma^d,\gamma^{d'}\}\mathbf{1}_{[\lambda<\lambda_1]}+\gamma^d\mathbf{1}_{[\lambda_1\leq\lambda<\lambda_2]}$. We have that the equilibrium under contextual ad targeting is more polarized than that under behavioral ad targeting if $\gamma>\gamma^{d''}$ and $1/3<\lambda<\lambda_2$. Moreover, $\gamma^{d''}<\gamma^m$ for all $\lambda\in(1/3,\lambda_2)$.

[3]Firm 2 has a stronger incentive to deviate to $x=1$ rather than $x=0$ to soften competition. So, we only need to consider its deviation to $x=1$.

2. The base utility $v \geq 1/2$: In this case, the consumer always derives a non-negative utility from consuming the content if the content location perfectly matches the consumer type. In addition, the consumer may consume the content even if the content location is on the opposite end of the consumer's location. One can show that $\overline{v} \geq 5/6$. The analyses are similar to the previous case.

■

Proof of Proposition 8. Based on the analyses of the main model, we need to show here that the additional utility term and the strategic choice of the consumers do not change the relative size of $\mathbf{Var}(\theta|x=0)$ and $\mathbf{Var}(\theta|x=1/2)$. We consider $M < 1/2$.

1. Suppose $x = 0$. Consider type $\theta = 0$ consumer first. We have $U(x,0) \geq 1/2 - M + \epsilon$. Hence,

$$\begin{aligned} & P(\text{a type 0 consumer consumes the content}) \\ \geq\ & P(1/2 - M + \epsilon \geq 0) \\ =\ & P(\epsilon \geq M - 1/2) \\ =\ & 1 - F(M - 1/2) \\ =\ & 1 - M \end{aligned}$$

Now consider type $\theta = 1/2$ consumer.

$$\begin{aligned} & P(\text{a type } 1/2 \text{ consumer consumes the content}) \\ \leq\ & P(1/2 - 1/2 - 0 + \epsilon \geq 0) \\ =\ & 1/2 \\ & P(\text{a type } 1/2 \text{ consumer consumes the content}) \\ \geq\ & P(1/2 - 1/2 - M + \epsilon \geq 0) \\ =\ & 1/2 - M \end{aligned}$$

Therefore,

$$\begin{aligned} & \mathbf{Var}(\theta|x=0,M) \\ =\ & \mathbf{E}[\theta^2|x=0,M] - \mathbf{E}[\theta|x=0,M]^2 \\ =\ & \frac{1}{4}\frac{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)}{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda) + P(\text{a type 0 consumer consumes})\lambda} - \\ & \left[\frac{1}{2}\frac{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)}{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda) + P(\text{a type 0 consumer consumes})\lambda}\right]^2 \\ =\ & \frac{1}{4}\frac{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)}{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda) + P(\text{a type 0 consumer consumes})\lambda}. \end{aligned}$$

$$
\begin{aligned}
&\frac{P(\text{a type 0 consumer consumes})\lambda}{P(\text{a type 1/2 consumer consumes})(1-2\lambda)+P(\text{a type 0 consumer consumes})\lambda} \\
\leq\ & \frac{1}{4}\frac{\frac{1}{2}(1-2\lambda)}{\frac{1}{2}(1-2\lambda)+(1-M)\lambda}\frac{\lambda}{(\frac{1}{2}-M)(1-2\lambda)+\lambda} \\
\overset{M\to 0}{\to}\ & \frac{1}{4}\frac{\frac{1}{2}(1-2\lambda)\lambda}{[\lambda+\frac{1}{2}(1-2\lambda)]^2} \\
=\ & \mathbf{Var}(\theta|x=0,M=0),
\end{aligned}
$$

which is the conditional variance in the base model.

2. Suppose $x = 1/2$. Similarly, we can show that in this case, $\mathbf{Var}(\theta|x = 1/2, M)$ is greater than or equal to an expression that converges to the conditional variance in the base model as $M \to 0$. According to Lemma 3, $\mathbf{Var}(\theta|x = 1/2) > \mathbf{Var}(\theta|x = 0)$ in the base model. Therefore, there exists $\hat{M} > 0$ such that $\mathbf{Var}(\theta|x = 1/2, M) > \mathbf{Var}(\theta|x = 0, M)$, for any $M \leq \hat{M}$. In such cases, the consumer's choice of niche content is privacy-reducing over the choice of mainstream content by definition. The same arguments as the ones in the main model imply that all the main insights of the monopoly case hold qualitatively if $M \leq \hat{M}$.

■

Proof of Proposition 9. We first show that the equilibrium number of firms located at $x = 0, N_0$ equals the equilibrium number of firms located at $x = 1, N_1$. Denote the equilibrium number of firms located at $x = 1/2$ by $N_{1/2}$. One can see that the expected ad revenues for firms located in the same position are the same. Denoted the expected ad revenue for a firm located at x as $r(x)$.

Suppose that $N_0 \neq N_1$. Without loss of generality, we assume that $N_0 > N_1 > 0$. It implies that each of the N_1 firms located at $x = 1$ obtains a zero profit, $r(1) - c = 0$. One can see that $r(0) \cdot N_0 = r(1) \cdot N_1$, which implies $r(0) < r(1)$. But then, we have $r(0) - c < r(1) - c = 0$. It will not be an equilibrium because a firm located at $x = 0$ would earn a negative profit and would deviate by exiting the market. Consequently, $N_0 = N_1$ in equilibrium.

Now we compare the equilibrium $\mathbf{Var}(\theta|x = 0)$ with $\mathbf{Var}(\theta|x = 1/2)$.

$$
\begin{aligned}
P(x=0|\theta=0) &= P(1/2+\epsilon_0 > 0+\epsilon_{1/2}) \\
&= P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&= 1 - F(0) + \int_{-1/2}^{0}\int_{-1/2}^{\epsilon_0+1/2} f(\epsilon_{1/2})d\epsilon_{1/2}f(\epsilon_0)d\epsilon_0 \\
&= 1 - F(0) + \int_{-1/2}^{0} F(\epsilon_0+1/2)f(\epsilon_0)d\epsilon_0 \\
&= 7/8, \\
P(x=0|\theta=1/2) &= P(0+\epsilon_0 > 1/2+\epsilon_{1/2})
\end{aligned}
$$

$$
\begin{aligned}
&= P(\epsilon_0 > \epsilon_{1/2} + 1/2) \\
&\overset{symmetry}{=} P(\epsilon_{1/2} > \epsilon_0 + 1/2) \\
&= 1 - P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&\overset{previous\ case}{=} 1 - 7/8 \ = 1/8
\end{aligned}
$$

$$
\begin{aligned}
&Privacy(\{0\}) \\
=&\mathbf{Var}[\theta|x=0] \\
=&\mathbf{E}[\theta^2|x=0] - \mathbf{E}[\theta|x=0]^2 \\
=&(\frac{1}{2})^2 \cdot P(\theta=1/2|x=0) + 1^2 \cdot P(\theta=1|x=0) - \left\{\frac{1}{2}\cdot P(\theta=1/2|x=0) + 1\cdot P(\theta=1|x=0)\right\}^2 \\
=&\frac{1}{4}\cdot\frac{P(x=0|\theta=1/2)P(\theta=1/2)}{P(x=0|\theta=1/2)P(\theta=1/2)+P(x=0|\theta=0)P(\theta=0)} + 1\cdot 0 - \\
&\left\{\frac{1}{2}\cdot\frac{P(x=0|\theta=1/2)P(\theta=1/2)}{P(x=0|\theta=1/2)P(\theta=1/2)+P(x=0|\theta=0)P(\theta=0)} + 1\cdot 0\right\}^2 \\
=&\frac{1}{4}\cdot\frac{\frac{1}{8}(1-2\lambda)}{\frac{1}{8}(1-2\lambda)+\frac{7}{8}\lambda} - \frac{1}{4}\cdot\left[\frac{\frac{1}{8}(1-2\lambda)}{\frac{1}{8}(1-2\lambda)+\frac{7}{8}\lambda}\right]^2 \\
=&\frac{1}{4}\frac{(1-2\lambda)(2+3\lambda)}{(1+5\lambda^2)}
\end{aligned}
$$

$$
\begin{aligned}
P(x=1/2|\theta=0) &= P(0+\epsilon_{1/2} > 1/2+\epsilon_0) \\
&= P(x=0|\theta=1/2) \\
&= 1/8 \\
P(x=1/2|\theta=1/2) &= P(1/2+\epsilon_{1/2} > 0+\epsilon_0 \ \&\ 1/2+\epsilon_{1/2} > 0+\epsilon_1) \\
&= P(1/2+\epsilon_{1/2} > \max\{\epsilon_0,\epsilon_1\}) \\
&= P(\epsilon_{1/2} \geq 0)\cdot 1 + P(\epsilon_{1/2} < 0 \ \&\ 1/2+\epsilon_{1/2} > \max\{\epsilon_0,\epsilon_1\}) \\
&= 1/2 + \int_{-1/2}^{0}\int_{-1/2}^{1/2+\epsilon_{1/2}} 2(m+1/2)\cdot 1 \ dm \ d\epsilon_{1/2} \\
&= 19/24,
\end{aligned}
$$

$$
\begin{aligned}
\text{where the cdf of } \max\{\epsilon_0,\epsilon_1\},\ F_m(m) &= P(\max\{\epsilon_0,\epsilon_1\} \leq m) \\
&= P(\epsilon_0 \leq m \ \&\ \epsilon_1 \leq m) \\
&\overset{independence}{=} P(\epsilon_0 \leq m)P(\epsilon_1 \leq m)
\end{aligned}
$$

$$= (m+1/2)^2,\ \forall m \in (-1/2, 1/2),$$

$$\text{so, the pdf of } \max\{\epsilon_0, \epsilon_1\},\ f_m(m) = F'_m(m) = 2(m+1/2),\ \forall m \in (-1/2, 1/2).$$

$$\begin{aligned}
&Privacy(\{1/2\}) \\
=&\mathbf{Var}[\theta | x = 1/2] \\
=&\mathbf{E}[(\theta - 1/2)^2 | x = 1/2] \\
=&(1/2)^2 \cdot P(\theta = 0 | x = 1/2) + \cdot (1/2)^2 \cdot P(\theta = 1 | x = 1/2) \\
=&2 \cdot (1/2)^2 \cdot P(\theta = 0 | x = 1/2) \\
=&\frac{1}{2} \cdot \frac{P(x = 1/2 | \theta = 0) P(\theta = 0)}{P(x = 1/2 | \theta = 0) P(\theta = 0) + P(x = 1/2 | \theta = 1) P(\theta = 1) + P(x = 1/2 | \theta = 1/2) P(\theta = 1/2)} \\
=&\frac{1}{2} \cdot \frac{P(x = 1/2 | \theta = 0) P(\theta = 0)}{2P(x = 1/2 | \theta = 0) P(\theta = 0) + P(x = 1/2 | \theta = 1/2) P(\theta = 1/2)} \\
=&\frac{1}{2} \cdot \frac{\frac{1}{8}\lambda}{2 \cdot \frac{1}{8}\lambda + \frac{5}{6}(1 - 2\lambda)} \\
=&\frac{3\lambda}{38 - 64\lambda}
\end{aligned}$$

$$\begin{aligned}
&Privacy(\{0\}) < Privacy(\{1/2\}) \\
\Leftrightarrow &\frac{1}{4} \frac{(1 - 2\lambda)(2 + 3\lambda)}{(1 + 5\lambda^2)} < \frac{3\lambda}{38 - 64\lambda} \\
\Leftrightarrow &g(\lambda) := -84\lambda^3 + 284\lambda^2 + 178\lambda - 76 > 0 \qquad (10)
\end{aligned}$$

Note that $g'(\lambda) = -252\lambda^2 + 568\lambda + 178,\ g''(\lambda) = -504\lambda + 568$. One can see that $g''(\lambda) > 0$ because $\lambda < 1/2$. Since $g'(1/3) > 0$, $g'(\lambda) > 0,\ \forall \lambda \in (1/3, 1/2)$. Since $g(1/3) > 0$, $g(\lambda) > 0,\ \forall \lambda \in (1/3, 1/2)$. Condition (10) always holds. Therefore, $Privacy(\{0\}) < Privacy(\{1/2\})$: the consumer's choice of niche content is privacy-reducing over the choice of mainstream content.

We now show that the ratio of the number of niche firms to the number of mainstream firms is higher under contextual ad targeting than under behavioral ad targeting.

Denote the number of niche firms located at $x = 0$ under behavioral ad targeting by N_0^b, the number of mainstream firms located at $x = 1/2$ under behavioral ad targeting by $N_{1/2}^b$, the number of niche firms located at $x = 0$ under contextual ad targeting by N_0^c, and the number of mainstream firms located at $x = 1/2$ under contextual ad targeting by $N_{1/2}^c$.

The market clearing condition implies that in equilibrium,

$$\begin{cases} D(0)/N_0^b - c = 0 \\ D(1/2)/N_{1/2}^b - c = 0 \\ D(0)[1-\gamma Privacy\{0\}]/N_0^c - c = 0 \\ D(1/2)[1-\gamma Privacy\{1/2\}]/N_{1/2}^b - c = 0 \end{cases}$$

$\Rightarrow N_0^c/N_{1/2}^c = N_0^b/N_{1/2}^b \cdot \frac{1-\gamma Privacy\{0\}}{1-\gamma Privacy\{1/2\}} > N_0^b/N_{1/2}^b$. ■

3 Extension with $K > 2$ Firms

Lemma 5. *Suppose there are M mainstream firms and N_0 and N_1 niche firms at ideological positions $x = 0$ and $x = 1$, respectively. The probability of a type-θ consumer choosing a media firm at position x is presented in the table below:*

Conditional Probability	*Value*
$P(x = 0 \mid \theta = 0)$	$\mathbb{1}(N_0 > 0)\left[1 - \mathbb{1}(M > 0)(1/2)^{N_0+M}\left(\sum_{k=1}^{M}\binom{M}{k}\frac{k}{N_0+k}\right)\right]$
$P(x = 1/2 \mid \theta = 0)$	$\mathbb{1}(M > 0)(1/2)^{N_0+M}\left(\sum_{k=1}^{M}\binom{M}{k}\frac{k}{N_0+k}\right)$
$P(x = 1 \mid \theta = 0)$	*0*
$P(x = 0 \mid \theta = 1/2)$	$\mathbb{1}(N_0 > 0)\frac{N_0}{N_0+N_1}(1/2)^{N_0+M+N_1}\left(\sum_{k=1}^{N_0+N_1}\binom{N_0+N_1}{k}\frac{k}{M+k}\right)$
$P(x = 1/2 \mid \theta = 1/2)$	$\mathbb{1}(M > 0)\left[1 - \mathbb{1}(N_0+N_1 > 0)(1/2)^{N_0+M+N_1}\left(\sum_{k=1}^{N_0+N_1}\binom{N_0+N_1}{k}\frac{k}{M+k}\right)\right]$
$P(x = 1 \mid \theta = 1/2)$	$\mathbb{1}(N_1 > 0)\frac{N_1}{N_0+N_1}(1/2)^{N_0+M+N_1}\left(\sum_{k=1}^{N_0+N_1}\binom{N_0+N_1}{k}\frac{k}{M+k}\right)$
$P(x = 0 \mid \theta = 1)$	*0*
$P(x = 1/2 \mid \theta = 1)$	$\mathbb{1}(M > 0)(1/2)^{N_1+M}\left(\sum_{k=1}^{M}\binom{M}{k}\frac{k}{N_1+k}\right)$
$P(x = 1 \mid \theta = 1)$	$\mathbb{1}(N_0 > 0)\left(1 - \mathbb{1}(M > 0)(1/2)^{N_1+M}\left(\sum_{k=1}^{M}\binom{M}{k}\frac{k}{N_1+k}\right)\right)$

Proof. Let $\epsilon_j^{(x)}$ denote the idiosyncratic term for the j^{th} firm at position x. We know that all idiosyncratic terms are independently drawn from $U[-1/2, 1/2]$.

We consider all three cases for θ.

1. *Case $\theta = 0$*: We know that a consumer with $\theta = 0$ will never choose $x = 1$, because $U(x = 1; \theta = 0) < 0$. So the choice is between N_0 niche firms at position $x = 0$ and M mainstream firms. The consumer receives utility $1/2 + \epsilon_j^{(0)}$ from choosing the firm at $x = 0$, and utility $\epsilon_i^{(1/2)}$ from choosing the firm at $x = 1/2$. As such, the consumer chooses a mainstream firm if there is at least one element in $\{\epsilon_i^{(1/2)}\}_{i=1}^{M}$ is greater than all elements in $\{1/2 + \epsilon_j^{(0)}\}_{j=1}^{N_0}$. We know that if any $\epsilon_j^{(0)} > 0$, there is a zero probability of a mainstream media firm being chosen. So for a mainstream firm to be chosen, we require all $\epsilon_j^{(0)}$'s to come from $[-1/2, 0]$, which has a probability of $(1/2)^{N_0}$. In that event, the mainstream firm k has a chance to

be chosen by the consumer only if $\epsilon_i^{(1/2)} \sim U[0, 1/2]$, i.e., the upper half of the uniform distribution. For any event where the error k out of M firms come from $U[0, 1/2]$ (i.e., $M-k$ firms with error from $U[-1/2, 0]$), the probability of one of these k values being the highest is $k/(k+N_0)$. We can characterize all such events with a Binomial distribution $B(M, 1/2)$, where the success probability refers to the error term being drawn from the upper half of $U[-1/2, 1/2]$. Therefore, we can write:

$$\begin{aligned} P(x = 1/2 \mid \theta = 0) &= (1/2)^{N_0} \left(\sum_{k=1}^{M} \binom{M}{k} (1/2)^k (1/2)^{M-k} \frac{k}{N_0 + k} \right) \\ &= (1/2)^{N_0+M} \left(\sum_{k=1}^{M} \binom{M}{k} \frac{k}{N_0 + k} \right) \end{aligned}$$

Based on the probability above, we can define the following probability:

$$P(x = 0 \mid \theta = 0) = 1 - (1/2)^{N_0+M} \left(\sum_{k=1}^{M} \binom{M}{k} \frac{k}{N_0 + k} \right)$$

2. *Case* $\theta = 1$: This case is conceptually the same as $\theta = 0$, but the difference is in the number of firms at position $x = 1$. We can use the logic presented in the case above to arrive at the following conditional probabilities:

$$\begin{aligned} P(x = 0 \mid \theta = 1) &= 0 \\ P(x = 1/2 \mid \theta = 1) &= (1/2)^{N_1+M} \left(\sum_{k=1}^{M} \binom{M}{k} \frac{k}{N_1 + k} \right) \\ P(x = 1 \mid \theta = 1) &= 1 - (1/2)^{N_1+M} \left(\sum_{k=1}^{M} \binom{M}{k} \frac{k}{N_1 + k} \right) \end{aligned}$$

3. *Case* $\theta = 1/2$: In this case, it is possible that the consumer chooses any niche type. The consumer receives utility $1/2 + \epsilon_i^{(1/2)}$ from choosing any mainstream firm. On the other hand, the consumer receives utility $\epsilon_j^{(0)}$ or $\epsilon_j^{(1)}$ from choosing firms at each niche position. Using the logic from previous cases, we can write the following probability for the union of events where consumer of type $\theta = 1/2$ chooses either the firm at $x = 0$ or $x = 1$:

$$\begin{aligned} P(x = 0 \vee x = 1 \mid \theta = 1/2) &= (1/2)^{M} \left(\sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} (1/2)^k (1/2)^{N_0+N_1-k} \frac{k}{M + k} \right) \\ &= (1/2)^{N_0+M+N_1} \left(\sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} \frac{k}{M + k} \right) \end{aligned}$$

The probability above is for the event where the consumer either chooses $x = 0$ or $x = 1$. Since the errors are independently drawn, the probability of choosing each niche position is proportional to the number of firms in that niche position. Hence, we can write the following probabilities for all three events:

$$P(x = 0 \mid \theta = 1/2) = \frac{N_0}{N_0 + N_1}(1/2)^{N_0+M+N_1} \left(\sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} \frac{k}{M + k} \right)$$

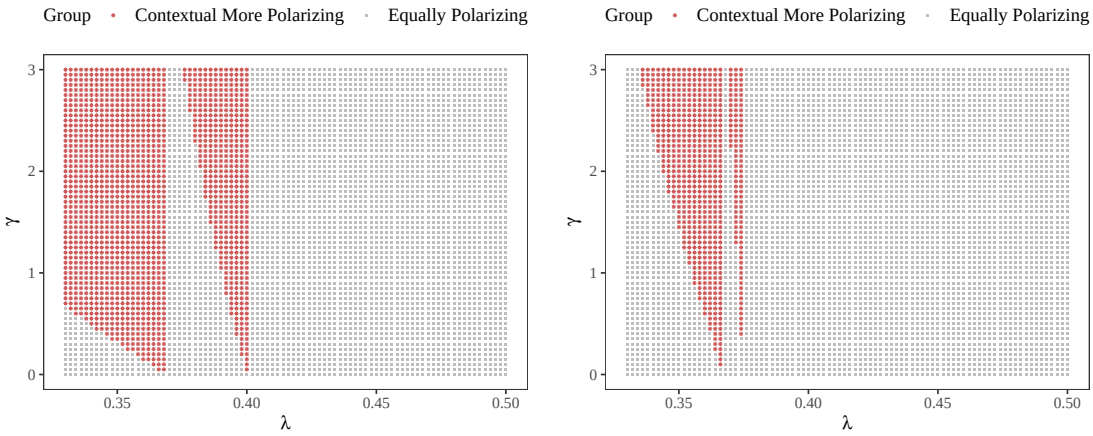
$$P(x = 1 \mid \theta = 1/2) = \frac{N_1}{N_0 + N_1}(1/2)^{N_0+M+N_1} \left(\sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} \frac{k}{M + k} \right)$$

$$P(x = 1/2 \mid \theta = 1/2) = 1 - (1/2)^{N_0+M+N_1} \left(\sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} \frac{k}{M + k} \right)$$

■

We now use the results from Lemma 5 to examine the equilibrium for cases with $K > 2$ media firms. Since we have a closed-form expression for conditional content choice probabilities, we can compute demand, conditional variance, and the total profit for each player in a (N_0, M, N_1) strategy profile under both contextual and behavioral targeting for a fixed set of λ and γ. This means that for a fixed set of λ and γ, we can enumerate over all (N_0, M, N_1) and assess if that strategy profile is an equilibrium by checking whether a player has an incentive to deviate. We know that the total number of profiles such that $N_0 + M + N_1 = K$ is equal to $\binom{K+2}{2}$, highlighting that the computational complexity of finding equilibria grows polynomially in the number of firms for fixed values of λ and γ.

We adopt this computational approach for $K = 3$ and $K = 4$ cases, and use a grid of 86 λ values in $[0.33, 0.50]$ with a 0.002 precision, and 61 γ values in $[0, 3]$ with a 0.05 precision. Figure 6 shows the cases in the parameter space where the equilibrium under contextual ad targeting is more, equally, and less polarizing than the equilibrium under behavioral ad targeting, for $K = 3$ (in Figure 6a) and for $K = 4$ (in Figure 6b). We call an equilibrium more polarizing if there is a higher proportion of firms in niche positions. Red points in these figures show the regions where the equilibrium is more polarizing under contextual ad targeting compared to behavioral ad targeting. This is the same as shaded region in Figure 5. As shown in both Figures 6a and 6b, our qualitative insight holds in these cases: under all possible combinations of (λ, γ) in our grid search, the equilibrium under contextual ad targeting is at least as polarizing as the equilibrium under behavioral ad targeting.



(a) Case with 3 Firms ($K = 3$)

(b) Case with 4 Firms ($K = 4$)

Figure 6: Comparison of equilibrium under contextual and behavioral ad targeting in cases with more than two firms when $\epsilon \sim U[-1/2, 1/2]$.

Note: Points in gray refer to cases where the equilibrium is equally polarizing under behavioral and contextual ad targeting. Red stars refer to cases where the equilibrium under contextual ad targeting is more polarizing that that under behavioral ad targeting (i.e., higher proportion of niche firms).