Online Appendix for Invitation to Search or Purchase? Optimal Multi-attribute Advertising

Yunfei (Jesse) Yao

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Proof of Proposition 6. One can see that the consumer will purchase the product for sure when $\mu_2 \geq \bar{\mu}(\mu_1)$ without advertising. So, the firm does not advertise when $\mu_2 \geq \bar{\mu}(\mu_1)$. Also, the consumer will not purchase the product when $\mu_1 \leq \underline{\mu}(1)$ if the firm does not advertise or advertises only one attribute, but will buy the product if the firm advertises both attributes and both turn out to be good. So, the firm advertises both attributes if $\mu_1 \leq \underline{\mu}(1)$. We now look at other cases.

(1) $\mu_1 > \underline{\mu}(1)$ and $\mu_2 \leq \underline{\mu}(1)$ (regions I_1 and I_2)

Proposition 5 has shown that the firm prefers advertising attribute two to advertising attribute one or not advertising. Therefore, we just need to compare the expected profits from advertising attribute two and from advertising both attributes. For a given prior belief (μ_1, μ_2), denote the consumer's probability of purchasing if the firm advertises both attributes by $P_b(\mu_1, \mu_2)$.

$$
\begin{aligned}
P_b(\mu_1, \mu_2) = &\ \mu_1 \cdot \mu_2, \\
P_2(\mu_1, \mu_2) = &\ \begin{cases} \mu_2 \cdot \frac{\mu_1 - \underline{\mu}(1)}{\bar{\mu}(1) - \underline{\mu}(1)},\ if\ \underline{\mu}(1) < \mu_1 < \bar{\mu}(1) \text{ (region } I_1) \\ \mu_2,\ if\ \mu_1 \geq \bar{\mu}(1) \text{ (region } I_2) \end{cases} .
\end{aligned}
$$

In region I_1, one can see that there exists $\mu^b \in (\underline{\mu}(1), \bar{\mu}(1))$ such that $P_b(\mu_1, \mu_2) > P_2(\mu_1, \mu_2)$ if $\mu_1 < \mu^b$ and $P_b(\mu_1, \mu_2) < P_2(\mu_1, \mu_2)$ if $\mu_1 > \mu^b$. In region I_2, one can see that advertising attribute two strictly dominates advertising both attributes.

(2) $\mu_1 \in (\underline{\mu}(1), \bar{\mu}(1)]$ and $\mu_2 > \underline{\mu}(1)$ (region I_3)

Similar to the previous case, we only need to compare the expected profits from advertising attribute two and from advertising both attributes. The same argument implies that $P_b(\mu_1, \mu_2) > P_2(\mu_1, \mu_2)$ if $\mu_1 < \mu^b$ and $P_b(\mu_1, \mu_2) < P_2(\mu_1, \mu_2)$ if $\mu_1 > \mu^b$.

(3) $\mu_1 > \bar{\mu}(1)$ and $\mu_2 \in (\underline{\mu}(1), \bar{\mu}(\mu_1))$ (region I_4, the diagonally striped black region, and the blank

search region)

Consider first region I_4. We just need to compare the expected profits from advertising attribute two and from advertising both attributes. Because $P_b(\mu_1, \mu_2) = \mu_1 \cdot \mu_2 < \mu_2 = P_2(\mu_1, \mu_2)$, advertising attribute two is still optimal in this case.

Consider then the diagonally striped black region. We just need to compare the expected profits from advertising attribute one and from advertising both attributes. Note that we have shown in Proposition 5 that advertising attribute one is better than advertising attribute two, $P_1(\mu_1, \mu_2) > P_2(\mu_1, \mu_2)$. Note also that $P_2(\mu_1, \mu_2) = \mu_2 > \mu_1 \cdot \mu_2 = P_b(\mu_1, \mu_2)$. So, $P_1(\mu_1, \mu_2) > P_2(\mu_1, \mu_2) > P_b(\mu_1, \mu_2)$.

Lastly, consider the blank search region. We just need to compare the expected profits from not advertising and from advertising both attributes. Note that we have shown in Proposition 5 that not advertising is better than advertising attribute one, $P(\mu_1, \mu_2) > P_1(\mu_1, \mu_2)$. Note also that $P_1(\mu_1, \mu_2) = \mu_1 > \mu_1 \cdot \mu_2 = P_b(\mu_1, \mu_2)$. So, $P(\mu_1, \mu_2) > P_1(\mu_1, \mu_2) > P_b(\mu_1, \mu_2)$.

The role of advertising follows from the consumer's strategy characterized in Section 3.

□

Proof of Proposition 7. If $\mu_2 \leq \mu_1 \leq \underline{\mu}(1)$, then one can see that the consumer's belief after seeing an ad about either attribute will be below the quitting boundary. So, the firm does not advertise. Also, the consumer will purchase the product for sure if $\mu_2 \geq \bar{\mu}(\mu_1)$ without advertising. So, the firm does not advertise if $\mu_2 \geq \bar{\mu}(\mu_1)$. We now look at other cases.

(1) μ_2 less than but close to $\bar{\mu}(\mu_1)$ (belief near the purchasing boundary)

(a) We first compare advertising attribute two with not advertising.

Suppose the firm advertises attribute two. With probability $\mu_2\gamma + (1-\mu_2)(1-\gamma)$, the consumer observes a good signal. In this case, the purchasing probability is at most one. With probability $\mu_2(1-\gamma)+(1-\mu_2)\gamma$, the consumer observes a bad signal. The consumer's belief about attribute two given a bad signal is $\mu_2^b = (1-\gamma)\mu_2/[(1-\gamma)\mu_2 + \gamma(1-\mu_2)]$.

The probability of not purchasing in this case is:

$$\mathbb{P}(\text{not purchasing}|s_2 = b) = \frac{\bar{\mu}(\mu_1) - \mu_2^b}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \wedge 1.$$

In sum, the probability of not purchasing if the firm advertises attribute two satisfies:

$$\begin{aligned}
&\mathbb{P}(\text{not purchasing}|\text{advertise attribute 2}) \\
=&\mathbb{P}(s_2 = g)\mathbb{P}(\text{not purchasing}|s_2 = g) + \mathbb{P}(s_2 = b)\mathbb{P}(\text{not purchasing}|s_2 = b) \\
\geq&\mathbb{P}(s_2 = b)\mathbb{P}(\text{not purchasing}|s_2 = b) \\
=&[\mu_2(1-\gamma) + (1-\mu_2)\gamma] \cdot \left[\frac{\bar{\mu}(\mu_1) - \mu_2^b}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \wedge 1\right].
\end{aligned}$$

The probability of not purchasing if the firm does not advertise is:

$$\mathbb{P}(\text{not purchasing}|\text{not advertise}) = \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)}.$$

Because $\mu_2(1-\gamma)+(1-\mu_2)\gamma \geq \min\{1-\mu_2, 1/2\} \geq \min\{1-\mu^{**}, 1/2\} := K \in (0, 1/2]$, a sufficient condition for $\mathbb{P}(\text{purchasing}|\text{advertise attribute 2}) \leq \mathbb{P}(\text{purchasing}|\text{not advertise}) \Leftrightarrow \mathbb{P}(\text{not purchasing}|\text{advertise attribute 2}) \geq \mathbb{P}(\text{not purchasing}|\text{not advertise})$ is:

$$K \cdot \left[\frac{\bar{\mu}(\mu_1) - \mu_2^b}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \wedge 1\right] \geq \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \tag{13}$$

If $\frac{\bar{\mu}(\mu_1)-\mu_2^b}{\bar{\mu}(\mu_1)-\underline{\mu}(\mu_1)} \geq 1$, then inequality (13) is equivalent to:

$$K \geq \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \Leftrightarrow \mu_2 \geq \bar{\mu}(\mu_1) - K[\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)].$$

Let $\widehat{\mu}_2'(\mu_1) = \bar{\mu}(\mu_1) - K[\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)]$. One can see that $\widehat{\mu}_2'(\mu_1) < \bar{\mu}(\mu_1)$ and that the firm prefers not advertising to advertising attribute two if $\mu_2 \geq \widehat{\mu}_2'(\mu_1)$.

If $\frac{\bar{\mu}(\mu_1)-\mu_2^b}{\bar{\mu}(\mu_1)-\underline{\mu}(\mu_1)} < 1$, then inequality (13) is equivalent to:

$$K \cdot \frac{\bar{\mu}(\mu_1) - \mu_2^b}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \geq \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \Leftrightarrow \mu_2 \geq \bar{\mu}(\mu_1) - \frac{1}{1 + \frac{1-K}{K} \frac{(1-\gamma)\frac{\mu_2}{1-\mu_2}+\gamma}{2\gamma-1}} \bar{\mu}(\mu_1) \tag{14}$$

Let $M(\mu_1) := [(1-\gamma)\bar{\mu}(\mu_1)/(1-\bar{\mu}(\mu_1)) + \gamma]/(2\gamma - 1) > 0$. One can see that:

$$\begin{aligned}
&\bar{\mu}(\mu_1) > \mu_2 \Rightarrow M(\mu_1) > \frac{(1-\gamma)\frac{\mu_2}{1-\mu_2} + \gamma}{2\gamma - 1} \\
\Rightarrow &\bar{\mu}(\mu_1) - \frac{1}{1 + \frac{1-K}{K} M(\mu_1)} \bar{\mu}(\mu_1) > \bar{\mu}(\mu_1) - \frac{1}{1 + \frac{1-K}{K} \frac{(1-\gamma)\frac{\mu_2}{1-\mu_2}+\gamma}{2\gamma-1}} \bar{\mu}(\mu_1)
\end{aligned}$$

Let $\widehat{\mu}_2'(\mu_1) = \bar{\mu}(\mu_1) - \frac{1}{1+\frac{1-K}{K}M(\mu_1)} \bar{\mu}(\mu_1) < \bar{\mu}(\mu_1)$. A sufficient condition for inequality (14) and thus inequality (13) to hold is $\mu_2 \geq \widehat{\mu}_2'(\mu_1)$. So, the firm prefers not advertising to advertising attribute two if $\mu_2 \geq \widehat{\mu}_2'(\mu_1)$.

(b) We then compare advertising attribute one with not advertising.

Suppose the firm advertises attribute one. With probability $\mu_1\gamma + (1 - \mu_1)(1 - \gamma)$, the consumer observes a good signal. In this case, the probability of purchasing is at most 1. With probability $\mu_1(1-\gamma)+(1-\mu_1)\gamma$, the consumer observes a bad signal. The consumer's belief about attribute one given a bad signal is $\mu_1^b = (1 - \gamma)\mu_1/[(1 - \gamma)\mu_1 + \gamma(1 - \mu_1)]$. We divide the problem into three cases.

i. $\mu_1^b > \mu_2$ and $\mu_1^b \geq \mu^{**}$.

$$\begin{aligned}
&\mathbb{P}(\text{purchasing}|s_1 = b) = P(\mu_1^b, \mu_2) = \frac{\mu_2 - \underline{\mu}(\mu_1^b)}{\bar{\mu}(\mu_1^b) - \underline{\mu}(\mu_1^b)} \vee 0 \\
\Rightarrow &\mathbb{P}(\text{not purchasing}|\text{advertise attribute 1}) \\
\geq &\mathbb{P}(\text{not purchasing}|s_1 = b)\mathbb{P}(s_1 = b) \\
= &[1 - \frac{\mu_2 - \underline{\mu}(\mu_1^b)}{\bar{\mu}(\mu_1^b) - \underline{\mu}(\mu_1^b)} \vee 0] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma] \\
= &[\frac{\bar{\mu}(\mu_1^b) - \mu_2}{\bar{\mu}(\mu_1^b) - \underline{\mu}(\mu_1^b)} \wedge 1] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma], \\
&\mathbb{P}(\text{not purchasing}|\text{not advertise}) = \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)}.
\end{aligned}$$

A sufficient condition for $\mathbb{P}(\text{purchasing}|\text{advertise attribute 1}) \leq \mathbb{P}(\text{purchasing}|\text{not advertise}) \Leftrightarrow$ $\mathbb{P}(\text{not purchasing}|\text{advertise attribute 1}) \geq \mathbb{P}(\text{not purchasing}|\text{not advertise})$ is:

$$[\frac{\bar{\mu}(\mu_1^b) - \mu_2}{\bar{\mu}(\mu_1^b) - \underline{\mu}(\mu_1^b)} \wedge 1] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma] \geq \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \tag{15}$$

According to Proposition 2, $\mu_1^b < \mu_1$ implies that $\bar{\mu}(\mu_1^b) > \bar{\mu}(\mu_1)$. One can see that the left-hand side of inequality (15) is strictly positive, whereas the right-hand side of inequality (15) is zero when $\mu_2 = \bar{\mu}(\mu_1)$. By continuity of the expressions in (15), there exists $\widehat{\mu}_1'(\mu_1) < \bar{\mu}(\mu_1)$ such that inequality (15) holds for any $\mu_2 \in [\widehat{\mu}_1'(\mu_1), \bar{\mu}(\mu_1)]$. Hence, the firm prefers not advertising to advertising attribute one when $\mu_2 \geq \widehat{\mu}_1'(\mu_1)$.

ii. $\mu_1^b > \mu_2$ and $\mu_1^b < \mu^{**}$.

$$\begin{aligned}
&\mathbb{P}(\text{purchasing}|s_1 = b) = P(\mu_1^b, \mu_2) = h(\mu_1^b, \mu_2)\tilde{P}(\mu_1^b) \\
\Rightarrow&\mathbb{P}(\text{not purchasing}|\text{advertise attribute 1}) \\
\geq&\mathbb{P}(\text{not purchasing}|s_1 = b)\mathbb{P}(s_1 = b) \\
=&[1 - h(\mu_1^b, \mu_2)\tilde{P}(\mu_1^b)] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma], \\
&\mathbb{P}(\text{not purchasing}|\text{not advertise}) = \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)}.
\end{aligned}$$

A sufficient condition for $\mathbb{P}(\text{purchasing}|\text{advertise attribute 1}) \leq \mathbb{P}(\text{purchasing}|\text{not advertise}) \Leftrightarrow$ $\mathbb{P}(\text{not purchasing}|\text{advertise attribute 1}) \geq \mathbb{P}(\text{not purchasing}|\text{not advertise})$ is:

$$\begin{aligned}
&[1 - h(\mu_1^b, \mu_2)\tilde{P}(\mu_1^b)] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma] \geq \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \\
\Leftrightarrow&\mu_2 \geq \bar{\mu}(\mu_1) - [\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)] \cdot [1 - h(\mu_1^b, \mu_2)\tilde{P}(\mu_1^b)] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma] \qquad (16)
\end{aligned}$$

Because $h(\mu_1^b, \mu_2) \leq 1$, we have:

$$\begin{aligned}
&\bar{\mu}(\mu_1) - [\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)] \cdot [1 - \tilde{P}(\mu_1^b)] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma] \\
\geq&\bar{\mu}(\mu_1) - [\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)] \cdot [1 - h(\mu_1^b, \mu_2)\tilde{P}(\mu_1^b)] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma]
\end{aligned}$$

Let $\widehat{\mu}_1'(\mu_1) = \bar{\mu}(\mu_1) - [\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)] \cdot [1 - \tilde{P}(\mu_1^b)] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma]$. One can see that inequality (16) holds, and thus the firm prefers not advertising to advertising attribute one, when $\mu_2 \geq \widehat{\mu}_1'(\mu_1)$. Because $\tilde{P}(\mu_1^b) < 1$ when $\mu_1^b < \mu^{**}$, we have $\widehat{\mu}_1'(\mu_1) < \bar{\mu}(\mu_1)$.

iii. $\mu_1^b < \mu_2$.

$$\begin{aligned}
&\mathbb{P}(\text{purchasing}|s_1 = b) = P(\mu_1^b, \mu_2) \overset{\mu_1^b < \mu_2}{<} P(\mu_2, \mu_2) = h(\mu_2, \mu_2)\tilde{P}(\mu_2) = 1 \cdot e^{-\int_{\mu_2}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx} \\
&\Rightarrow \mathbb{P}(\text{not purchasing}|\text{advertise attribute 1}) \\
&\geq \mathbb{P}(\text{not purchasing}|s_1 = b)\mathbb{P}(s_1 = b) \\
&> [1 - e^{-\int_{\mu_2}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx}] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma], \\
&\mathbb{P}(\text{not purchasing}|\text{not advertise}) = \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)}.
\end{aligned}$$

A sufficient condition for $\mathbb{P}(\text{purchasing}|\text{advertise attribute 1}) \leq \mathbb{P}(\text{purchasing}|\text{not advertise}) \Leftrightarrow \mathbb{P}(\text{not purchasing}|\text{advertise attribute 1}) \geq \mathbb{P}(\text{not purchasing}|\text{not advertise})$ is:

$$[1 - e^{-\int_{\mu_2}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx}] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma] \geq \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \tag{17}$$

Because $1 - e^{-\int_{\mu_2}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx} > 1 - e^{-\int_{\bar{\mu}(\mu_1)}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx}$, a sufficient condition for inequality (17) to hold is:

$$\begin{aligned}
&[1 - e^{-\int_{\bar{\mu}(\mu_1)}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx}] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma] \geq \frac{\bar{\mu}(\mu_1) - \mu_2}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)} \\
&\Leftrightarrow \mu_2 \geq \bar{\mu}(\mu_1) - [\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)] \cdot [1 - e^{-\int_{\bar{\mu}(\mu_1)}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx}] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma].
\end{aligned}$$

Let $\widehat{\mu}_1'(\mu_1) = \bar{\mu}(\mu_1) - [\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)] \cdot [1 - e^{-\int_{\bar{\mu}(\mu_1)}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx}] \cdot [\mu_1(1-\gamma) + (1-\mu_1)\gamma] < \bar{\mu}(\mu_1)$. One can see that the firm prefers not advertising to advertising attribute one if $\mu_2 \geq \widehat{\mu}_1'(\mu_1)$.

Let $\widehat{\mu}'(\mu_1) := \max\{\widehat{\mu}_1'(\mu_1), \widehat{\mu}_2'(\mu_1)\}$. One can see that $\widehat{\mu}'(\mu_1) < \bar{\mu}(\mu_1)$ and that the firm does not advertise if $\widehat{\mu}'(\mu_1) \leq \mu_2 < \bar{\mu}(\mu_1)$.

(2) $\mu_1 > \underline{\mu}(1)$ and $\mu_2 \leq \underline{\mu}(1)$ (regions I_1 and I_2)

The consumer will never purchase the product if the firm advertises attribute one or does not advertise. In contrast, the consumer may purchase the product if the firm advertises on attribute two. The consumer will not purchase if the signal is bad. So, we focus on the case in which the signal is good. Advertising attribute 2 is strictly better than not advertising if and only if the posterior belief is above the quitting boundary when the signal is good:

$$\begin{aligned} &\mathbb{P}(\text{attribute 2 is good}|s_2 = g) > \underline{\mu}(\mu_1) \\ \Leftrightarrow &\frac{\gamma\mu_2}{\gamma\mu_2 + (1-\gamma)(1-\mu_2)} > \underline{\mu}(\mu_1) \end{aligned} \tag{18}$$

One can see that the left-hand side of equation (18) is increasing in γ and μ_2, and that the right-hand side of equation (18) is decreasing in μ_1. Notice that the inequality holds if $\gamma = 1$ and does not hold if $\gamma = 1/2$. Therefore, there exists $\hat{\gamma}(\mu_1, \mu_2) \in (1/2, 1)$, which is decreasing in μ_1 and μ_2, such that the firm advertises attribute two if $\gamma > \hat{\gamma}(\mu_1, \mu_2)$ and does not advertise if $\gamma < \hat{\gamma}(\mu_1, \mu_2)$.

□

Proof of Proposition 8. Consider first manipulating μ_1. By incurring cost $C(\Delta\mu)$, the firm can increase the initial belief about attribute one from μ_1 to $\mu_1 + \Delta\mu$. According to Proposition 5, it is optimal for the firm to advertise attribute two after belief manipulation. One can see that the purchasing probability does not increase in μ_1 once the belief reaches the boundary of region I_2. Thus, the firm never increases μ_1 above $\bar{\mu}(1)$, $\Delta\mu \in [0, \bar{\mu}(1) - \mu_1]$. In such cases, one can see that the purchasing probability given belief (μ_1, μ_2) is:

$$P_2(\mu_1, \mu_2) = \mu_2 \cdot \frac{\mu_1 - \underline{\mu}(1)}{\bar{\mu}(1) - \underline{\mu}(1)}.$$

Therefore, the marginal benefit of increasing a unit of μ_1 is $(p-m)\mu_2/[\bar{\mu}(1)-\underline{\mu}(1)]$. If $C'(\bar{\mu}(1)-\mu_1) > (p-m)\mu_2/[\bar{\mu}(1)-\underline{\mu}(1)]$, then the optimal $\Delta\mu$ will be smaller than $\bar{\mu}(1)-\mu_1$ and thus the manipulated belief will be located in region I_1. If $C'(\bar{\mu}(1) - \mu_1) \leq (p - m)\mu_2/[\bar{\mu}(1) - \underline{\mu}(1)]$, then, the optimal $\Delta\mu$ will be $\bar{\mu}(1) - \mu_1$ and thus the manipulated belief will be $(\bar{\mu}(1), \mu_2)$.[1]

[1] We are analyzing the optimal targeting strategy and manipulated belief *conditional on* manipulating μ_1. It may

Now consider manipulating μ_2. By incurring cost $C(\Delta\mu)$, the firm can increase the initial belief about attribute two from μ_2 to $\mu_2 + \Delta\mu$. According to Proposition 5 and by symmetry, it is optimal for the firm to advertise attribute one if the manipulated belief $\mu_2 + \Delta\mu < \tilde{\mu}^{-1}(\max\{\mu_1, \tilde{\mu}(1)\})$, and to advertise attribute two if the manipulated belief $\mu_2 + \Delta\mu > \tilde{\mu}^{-1}(\max\{\mu_1, \tilde{\mu}(1)\})$. Similar to the previous argument, the marginal benefit of increasing a unit of μ_2 is $(p-m)\mu_1/[\bar{\mu}(1) - \underline{\mu}(1)]$ if $\mu_2 + \Delta\mu \leq \bar{\mu}(1)$, 0 if $\mu_2 + \Delta\mu \in (\bar{\mu}(1), \tilde{\mu}^{-1}(\max\{\mu_1, \tilde{\mu}(1)\}))$, and $(p-m)(\mu_1 - \underline{\mu}(1))/[\bar{\mu}(1) - \underline{\mu}(1)]$ if $\mu_2 + \Delta\mu \geq \tilde{\mu}^{-1}(\max\{\mu_1, \tilde{\mu}(1)\})$. As a result, as long as the screening cost is not too low, $C'(\tilde{\mu}^{-1}(\max\{\mu_1, \tilde{\mu}(1)\}) - \mu_2) \geq (p-m)\mu_1/[\bar{\mu}(1) - \underline{\mu}(1)]$, the firm never increases μ_2 above $\bar{\mu}(1)$.

If $C'(\bar{\mu}(1) - \mu_2) > (p-m)\mu_1/[\bar{\mu}(1) - \underline{\mu}(1)]$, then the optimal $\Delta\mu$ will be smaller than $\bar{\mu}(1) - \mu_2$ and thus the manipulated belief will be located in region I_1 or I_3. If $C'(\bar{\mu}(1) - \mu_2) \leq (p-m)\mu_1/[\bar{\mu}(1) - \underline{\mu}(1)]$, then, the optimal $\Delta\mu$ will be $\bar{\mu}(1) - \mu_2$ and thus the manipulated belief will be $(\mu_1, \bar{\mu}(1))$.[2]

Putting the above together, if the screening cost is high, $C'(\bar{\mu}(1) - \mu_1) > (p-m)\mu_2/[\bar{\mu}(1) - \underline{\mu}(1)]$ and $C'(\bar{\mu}(1) - \mu_2) > (p-m)\mu_1/[\bar{\mu}(1) - \underline{\mu}(1)]$ (which implies $C'(\tilde{\mu}^{-1}(\max\{\mu_1, \tilde{\mu}(1)\}) - \mu_2) \geq (p-m)\mu_1/[\bar{\mu}(1) - \underline{\mu}(1)]$ because of the convexity of $C(\cdot)$), then the manipulated belief given the optimal targeting activity will be located in region I_1 or I_3. The role of optimal advertising is to invite consumers to search. If the screening cost is low, $C'(\bar{\mu}(1) - \mu_1) \leq (p-m)\mu_2/[\bar{\mu}(1) - \underline{\mu}(1)]$ and $C'(\bar{\mu}(1) - \mu_2) \leq (p-m)\mu_1/[\bar{\mu}(1) - \underline{\mu}(1)] \leq C'(\tilde{\mu}^{-1}(\max\{\mu_1, \tilde{\mu}(1)\}) - \mu_2)$, then the manipulated belief given the optimal targeting activity will be either $(\bar{\mu}(1), \mu_2)$ or $(\mu_1, \bar{\mu}(1))$. The role of optimal advertising is to invite consumers to purchase.

$\square$

be optimal for the firm to manipulate μ_2 instead.

[2] We are analyzing the optimal targeting strategy and manipulated belief *conditional on* manipulating μ_2. It may be optimal for the firm to manipulate μ_1 instead.