# Online Appendix for Privacy and Polarization: An Inference-Based Framework

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## 1 Ad Revenue Under a Competitive Auction Environment

In the main text, we consider a case where the media firm chooses each ad and extracts the match value as revenue. In reality, however, the media firm does not often have this level of control over ad allocation because advertising slots are sold through auctions where advertisers compete for impressions. These auctions are generally sold by ad intermediaries and platforms such as Google Ads, and the intermediary collects an $\alpha$ proportion (e.g., 30%) of the total revenue extracted from advertisers. In this section, we show that if the media firm delegates this decision to an auctioneer and allows ads to compete in an auction environment, the model setup remains unchanged.

**Auction Environment:** Suppose there is a large number of infinitesimal ad campaigns located on $[0, 1]$ who compete in an auction environment. Following the convention of this literature, the auctioneer runs a standard auction, such as a second- or first-price auction. We focus on the second-price auction in our analysis here, but one can show the revenue equivalence for the case of first-price (Myerson, 1981).

**Advertisers' Bidding Behavior:** Each ad $a$ receives value $1 - \gamma(a - \theta)^2$ from being shown to a type-$\theta$ consumer. However, advertisers do not necessarily know the consumer's type and need to form beliefs about it given the information available. As such, their ex-ante value of an impression given information $\mathcal{I}$ about that impression is $\mathbf{E}[1 - \gamma(a - \theta)^2 \mid \mathcal{I}]$. Under behavioral targeting, this ex-ante valuation is $1 - \gamma(a - \theta)^2$ because the advertiser knows the consumer's type $\theta$. However, under contextual targeting, the ex-ante valuation for an impression is $1-\gamma\mathbf{E}[(a-\theta)^2 \mid x]$ because the information available is only the consumer's content choice $x$. Because advertisers are competing in a second-price auction where truth-telling is the equilibrium bidding behavior, each ad $a$ submits the bid $1 - \gamma\mathbf{E}[(a - \theta)^2 \mid \mathcal{I}]$ depending on the information available.

**Expected Ad Revenue:** In a second-price auction with infinitesimal ad campaigns, the impression is allocated to the ad with the highest bid, that is, $a^* = \arg\max_a 1 - \gamma\mathbf{E}[(a - \theta)^2 \mid \mathcal{I}]$. One could easily use the main property of variance and show that $a^* = \mathbf{E}[\theta \mid \mathcal{I}]$ (a formal proof is provided in Lemma 1). Because there are multiple bidders at $a^*$, the auctioneer can extract all the value as the total ad revenue for the impression with information $\mathcal{I}$ under the second-price auction. We define the firm's ad revenue for an impression with information $\mathcal{I}$ as $\text{AdRev}(\mathcal{I})$ as follows:

$$\begin{aligned}\text{AdRev}(\mathcal{I}) &= (1-\alpha)\left(1 - \gamma\mathbf{E}\left[(\mathbf{E}[\theta \mid x] - \theta)^2 \mid \mathcal{I}\right]\right)\\ &= (1-\alpha)\left(1 - \gamma\mathbf{Var}(\theta \mid \mathcal{I})\right),\end{aligned}$$

where $(1-\alpha)$ is the share of total revenue that is collected by the media firm.

**Media Firm's Profit Maximization:** Based on the ad revenue collected through auctions in a setting where the media firm does not have full control over ad allocation, the firm's profit maximization problem can be written as follows:

$$\max_x D(x)\text{AdRev}(\mathcal{I})$$

It is easy to show that this maximization problem is equivalent to the one we use in the main text as shown in Corollary 1. Therefore, the media firm's full control over ad allocation is not a requirement for our main results and we will arrive at the same insights if advertisers can self-select into impressions in an auction environment.

In light of this equivalence, one could view the value of inference from the point-of-view of advertisers in a market environment. If consumers' self-selection into content provides sharper inference about the consumer type, this information will be reflected in advertisers' bids, which translates into higher revenues for the media firm.

## 2 Proofs

*Proof of Lemma 4.* There are three candidate equilibria: (0,1/2), (1/2,1/2), (0,1).

1. Under (0,1) strategy profile, the demands for type 0 and type 1 content are symmetric. So, we only need to examine type 0 content. Each consumer will choose type 0 content if and only if the utility from type 0 content is positive (higher than the outside option) and higher than the utility from type 1 content.

   One can see that all type 0 consumers will choose type 0 content. Now consider type 1/2 consumers. A mainstream consumer will consume one of the contents if and only if the utility from at least one content is positive. By symmetry, the consumer's overall probabilities of choosing type 0 and type 1 content are identical.

$$\begin{aligned}
&P(\text{a mainstream consumer chooses type 0 content}) \\
&= \frac{1}{2} \cdot P(\max\{\epsilon_0, \epsilon_1\} > 0) \\
&\overset{\textit{independence of } \epsilon_j}{=} \frac{1}{2} \cdot [1 - P(\epsilon_0 \le 0)P(\epsilon_1 \le 0)] \\
&= \frac{1}{2} \cdot [1 - F(0)^2]
\end{aligned}$$

   In sum, the demand of type 0 content from type 0 consumers is $\lambda$ and from type 1/2 consumers is $[1-F(0)^2](1-2\lambda)/2$. The total demand of type 0 content is $\lambda + [1-F(0)^2](1-2\lambda)/2$.

2. Under (1/2, 1/2) strategy profile, one can see that all mainstream consumers will consume the content by one of the media firms. The demands from type 0 and type 1 consumers are symmetric. Consider type 0 consumers. They will consume the content by one of the firms if and only if their utility from at least one content is positive. By symmetry, their overall probabilities of choosing either mainstream content are identical.

$$\begin{aligned} & P(\text{a type 0 consumer chooses firm } i) \\ = {} & \frac{1}{2} \cdot P(\max\{\epsilon_{1/2}, \epsilon_{1/2'}\} > 0) \\ \overset{\textit{independence of } \epsilon_j}{=} {} & \frac{1}{2} \cdot [1 - P(\epsilon_{1/2} \leq 0) P(\epsilon_{1/2'} \leq 0)] \\ = {} & \frac{1 - F(0)^2}{2} \end{aligned}$$

In sum, the demand of either firm from mainstream consumers is $(1-2\lambda)/2$, from type 0 consumers is $[1 - F(0)^2]\lambda/2$, and from type 1 consumers is $\frac{1-F(0)^2}{2}\lambda$. The total demand of either firm is $(1-2\lambda)/2 + [1 - F(0)^2]\lambda/2$.

3. Under (0, 1/2) strategy profile, consider first the demand for type 0 content. Since type 0 consumers' utility from consuming type 0 content is always positive, we only need to compare their utility from type 0 and type 1/2 content.

$$\begin{aligned} & P(\text{a type 0 consumer chooses type 0 content}) \\ = {} & P(1/2 + \epsilon_0 > 0 + \epsilon_{1/2}) \\ = {} & P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\ = {} & 1 - F(0) + \int_{-1/2}^{0} \int_{-1/2}^{\epsilon_0 + 1/2} f(\epsilon_{1/2}) d\epsilon_{1/2} f(\epsilon_0) d\epsilon_0 \\ = {} & 1 - F(0) + \int_{-1/2}^{0} F(\epsilon_0 + 1/2) f(\epsilon_0) d\epsilon_0 \\ \overset{\text{integral by parts}}{=} {} & 1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0 \end{aligned}$$

A type 1/2 consumer will choose type 0 content if and only if the utility of consuming it is positive and higher than the utility of consuming mainstream content. Since the utility of consuming mainstream content is always positive, we only need the condition that the utility of consuming type 0 content is higher than the utility of consuming mainstream content.

$$\begin{aligned} & P(\text{a type 1/2 consumer chooses type 0 content}) \\ = {} & P(0 + \epsilon_0 > 1/2 + \epsilon_{1/2}) \end{aligned}$$

$$
\begin{aligned}
&= P(\epsilon_0 > \epsilon_{1/2} + 1/2) \\
&\overset{symmetry}{=} P(\epsilon_{1/2} > \epsilon_0 + 1/2) \\
&= 1 - P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&\overset{previous\ case}{=} 1 - [1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] \\
&= \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

Therefore, the demand of type 0 content from type 0 consumers is $\lambda[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0]$ and from type 1/2 consumers is $(1-2\lambda) \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0+1/2) d\epsilon_0$. The total demand of of type 0 content is $\lambda[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] + (1 - 2\lambda) \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$.

Consider now the demand for type 1/2 content.

$$
\begin{aligned}
&P(\text{a type 0 consumer chooses type 1/2 content}) \\
&= 1 - P(\text{a type 0 consumer chooses type 0 content}) \\
&= \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

$$
\begin{aligned}
&P(\text{a type 1/2 consumer chooses type 1/2 content}) \\
&= 1 - P(\text{a type 1/2 consumer chooses type 0 content}) \\
&= 1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}
$$

$$
\begin{aligned}
&P(\text{a type 1 consumer chooses type 1/2 content}) \\
&= P(U(1/2, 1) > 0) \\
&= P(\epsilon_{1/2} > 0) \\
&= 1 - F(0)
\end{aligned}
$$

Therefore, the demand of type 1/2 content from type 0 consumers is $\lambda \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$, from type 1/2 consumers is $(1 - 2\lambda)[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0]$, and from type 1 consumers is $[1 - F(0)]\lambda$. The total demand of type 1/2 content is $\lambda \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0 + (1 - 2\lambda)[1 - \int_{-1/2}^{0} F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] + [1 - F(0)]\lambda$.

■

*Proof of Proposition 6.* We first compute the consumer utility from content consumption for each possible equilibrium strategy profile.

1. Under (0,1) strategy profile, the consumer utility from consuming each firm's content is:

$$\begin{aligned}&\lambda(1/2+\mathbf{E}[\epsilon_0])+(3/8-3\lambda/4)\mathbf{E}[\epsilon_0|\epsilon_0>0 \text{ and } \epsilon_0>\epsilon_1]\\=&\lambda/2+(3/8-3\lambda/4)\cdot 5/18=\frac{5-\lambda}{18}\end{aligned}$$

   Hence, the total consumer utility from content consumption is $2(5-\lambda)/18=(5-\lambda)/9$.

2. Under (1/2,1/2) strategy profile, the consumer utility from consuming each firm's content is:

$$\begin{aligned}&\frac{1-2\lambda}{2}(1/2+\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>\epsilon_{1/2'}])+2(\frac{1-F(0)^2}{2})\lambda\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0 \text{ and } \epsilon_{1/2}>\epsilon_{1/2'}]\\=&\frac{1-2\lambda}{3}+\frac{3\lambda}{4}\frac{5}{18}=\frac{8-11\lambda}{24}\end{aligned}$$

   Hence, the total consumer utility from content consumption is $2(6-7\lambda)/24=(6-7\lambda)/12$.

3. Under (0,1/2) strategy profile, the consumer utility from consuming firm 1's content is:

$$\begin{aligned}&\frac{7\lambda}{8}(1/2+\mathbf{E}[\epsilon_0|\epsilon_0+1/2>\epsilon_{1/2}])+\frac{1-2\lambda}{8}\mathbf{E}[\epsilon_0|\epsilon_0>0 \text{ and } \epsilon_0>1/2+\epsilon_{1/2}]\\=&\frac{7\lambda}{8}\cdot(\frac{1}{2}+\frac{1}{21})+\frac{1-2\lambda}{8}\cdot\frac{1}{3}=\frac{2+19\lambda}{48}\end{aligned}$$

   The consumer utility from consuming firm 2's content is:

$$\begin{aligned}&\frac{\lambda}{8}\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0 \text{ and } \epsilon_{1/2}>1/2+\epsilon_0]+\frac{7(1-2\lambda)}{8}(1/2+\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>1/2+\epsilon_0)+\\&\frac{\lambda}{2}\mathbf{E}[\epsilon_{1/2}|\epsilon_{1/2}>0]\\=&\frac{\lambda}{8}\cdot\frac{1}{3}+\frac{7(1-2\lambda)}{8}(1/2+1/3)+\frac{\lambda}{2}\cdot\frac{1}{4}=\frac{35-62\lambda}{48}\end{aligned}$$

   Hence, the total consumer utility from content consumption is $(2+19\lambda)/48+(35-62\lambda)/48=(37-43\lambda)/48$.

The total consumer utility from content consumption under (0,1) strategy profile is higher than that under (0,1/2) strategy profile if and only if $2\cdot(5-\lambda)/18>(37-43\lambda)/48 \Leftrightarrow \lambda>31/113$, which always holds. The total consumer utility from content consumption under (0,1/2) strategy profile is higher than that under (1/2,1/2) strategy profile if and only if $(37-43\lambda)/48>2(8-11\lambda)/24 \Leftrightarrow \lambda>-5$, which always holds. ■

*Proof of Proposition 7.* We consider two cases:

1. The base utility $v < 1/2$ : In this case, the consumer may not consume the content even if the content location perfectly match the consumer type. In addition, the consumer never consumes the content if the content location is on the opposite end of the consumer's location.

   (a) Monopoly

   We first summarize the demand for a monopoly by choosing a particular content.

   Suppose the monopoly chooses niche content strategy $x = 0$.[1] One can show that the demand from type 0 consumer is $\lambda(1/2 + v)$, from type 1/2 consumer is $(1 - 2\lambda)v$, and from type 1 consumer is 0. Hence, the total demand is $\lambda(1/2 + v) + (1 - 2\lambda)v$.

   Suppose the monopoly chooses mainstream content strategy $x = 1/2$. One can show that the demand from type 0 consumer is $\lambda v$, from type 1/2 consumer is $(1 - 2\lambda)(1/2 + v)$, and from type 1 consumer is $\lambda v$. Hence, the total demand is $(1 - 2\lambda)(1/2 + v) + 2\lambda v$.

$$D(0) < D(1/2) \Leftrightarrow \lambda < \frac{1}{3 - 2v}$$

   Consequently, the monopoly chooses mainstream positioning under behavioral ad targeting if and only if $\lambda < 1/(3 - 2v)$.

   Now let us study the equilibrium under contextual ad targeting. We restrict the attention to the case where $1/3 < \lambda < 1/(3 - 2v)$ because niche positioning gives the firm higher demand and better inference if $\lambda \geq 1/(3-2v)$ whereas mainstream positioning dominates niche positioning if $\lambda \leq 1/3$.

   Some calculations yield that $\mathbf{Var}(\theta|x = 0) = v(1-2\lambda)(\lambda v+\lambda/2)/[4(\lambda/2-\lambda v+v)^2]$ and $\mathbf{Var}(\theta|x = 1/2) = \lambda v/(1 + 2v - 2\lambda)$. Niche positioning gives the firm better inference if and only if $\mathbf{Var}(\theta|x = 0) < \mathbf{Var}(\theta|x = 1/2)$, which always hold for $\lambda \in (1/3, 1/(3-2v))$. The monopoly prefers niche position to mainstream position if:

$$\begin{aligned}
&\pi_{con}(x = 0) > \pi_{con}(x = 1/2) \\
\Leftrightarrow &D(0)[1 - \gamma\mathbf{Var}(\theta|x = 0) > D(1/2)[1 - \gamma\mathbf{Var}(\theta|x = 1/2)] \\
\Leftrightarrow &\gamma > \frac{16[1 + \lambda(2v - 3)]}{\lambda v[2\lambda^3(1 - 2v)^2(1 + 2v) + 4\lambda v(-1 + 2v + 8v^2) + \lambda^2(-1 + 10v + 4v^2 - 40v^3) - 4(-4 + v^2 + 2v^3)]}
\end{aligned}$$

   Denote the threshold as $\gamma^m$. One can show that $\gamma^m$ decreases in $\lambda$ and increases in $v$.[2]

   Now consider consumer utility from content consumption. Denote the consumer utility from content consumption given content strategy $x$ as $CW(x)$. We have:

$$\begin{aligned}
CW(1/2) =&(1 - 2\lambda)\mathbf{E}[v + \epsilon|v + \epsilon > 0]P(v + \epsilon > 0)+ \\
&2\lambda\mathbf{E}[v - 1/2 + \epsilon|v - 1/2 + \epsilon > 0]P(v - 1/2 + \epsilon > 0)
\end{aligned}$$

[1] The case where $x = 1$ is symmetric to this case. We only need to consider one of these cases.

[2] Observe that $\gamma^m$ can be negative for small $v$. In that case, we let $\gamma^m = 0$ for convenience.

$$=\frac{(1-2\lambda)(v+1/2)^2}{2}+\lambda v^2$$

$$CW(0) = \lambda\mathbf{E}[v+\epsilon|v+\epsilon>0]P(v+\epsilon>0)+$$
$$(1-2\lambda)\mathbf{E}[v-1/2+\epsilon|v-1/2+\epsilon>0]P(v-1/2+\epsilon>0)$$
$$=\frac{\lambda(v+1/2)^2}{2}+\frac{(1-2\lambda)v^2}{2}$$

Niche content strategy leads to lower consumer welfare if: $CW(0) < Cw(1/2) \Leftrightarrow \lambda < \hat{\lambda} := (v+1/4)/(-v^2+3v+1/4)$. One can see that $\hat{\lambda} > 1/3$.

(b) Duopoly

We first compute the demand given different strategy profiles.

i. Under (0,1) strategy profile, the demand for type 0 content:
From $\theta = 0$ consumers: $\lambda P(v+\epsilon>0) = \lambda(v+1/2)$.
From $\theta = 1/2$ consumers: $(1/2)(1-2\lambda)P(\max\{v-1/2+\epsilon_0, v-1/2+\epsilon_1\}>0) = (1-2\lambda)(2v-v^2)/2$.
From $\theta = 1$ consumers: 0.
Total demand: $\lambda(v+1/2)+(1-2\lambda)(2v-v^2)/2$.
The demand for type 1 content is symmetric.

ii. Under (1/2, 1/2) strategy profile, the demand for type 1/2 content:
From $\theta = 0/1$ consumers: $(1/2)\lambda P(\max\{v-1/2+\epsilon_{1/2}, v-1/2+\epsilon_{1/2'}\}>0) = \lambda(2v-v^2)/2$.
From $\theta = 1/2$ consumers: $(1/2)(1-2\lambda)P(\max\{v+\epsilon_{1/2}, v+\epsilon_{1/2'}\}>0) = (1-2\lambda)[1-(1/2-v)^2]/2$.
Total demand: $\lambda(2v-v^2)+(1-2\lambda)[1-(1/2-v)^2]/2$.

iii. Under (0,1/2) strategy profile, the demand for type 0 content:
From $\theta = 0$ consumers: $\lambda P(v+\epsilon_0>0 \text{ and } v+\epsilon_0>v-1/2+\epsilon_{1/2}) = \lambda(v/2+5/8)$.
From $\theta = 1/2$ consumers: $(1/2)\lambda P(v-1/2+\epsilon_0>0 \text{ and } v-1/2+\epsilon_0>v+\epsilon_{1/2}) = (1-2\lambda)(v-v^2)/2$.
From $\theta = 1$ consumers: 0.
Total demand: $\lambda(v/2+5/8)+(1-2\lambda)(v-v^2)/2$.

The demand for type 1/2 content:
From $\theta = 0$ consumers: $\lambda(v-v^2)/2$.
From $\theta = 1/2$ consumers: $(1-2\lambda)(v/2+5/8)$.
From $\theta = 1$ consumers: $\lambda P(v-1/2+\epsilon_{1/2}>0) = \lambda v$.
Total demand: $\lambda(v-v^2)/2+(1-2\lambda)(v/2+5/8)+\lambda v$.

Consider the equilibrium under behavioral ad targeting. Suppose (0,1/2) is an equilibrium. Firm 1 will deviate from $x = 0$ to $x = 1/2$ if and only if:

$$\begin{aligned}
&\lambda(2v - v^2) + \frac{(1-2\lambda)[1-(1/2-v)^2]}{2} > \lambda(v/2 + 5/8) + \frac{(1-2\lambda)(v-v^2)}{2} \\
\Leftrightarrow &\lambda < \lambda_2 := \frac{3}{11 - 12v + 8v^2} (< \frac{1}{3-2v})
\end{aligned}$$

Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:

$$\begin{aligned}
&\lambda(v + 1/2) + \frac{(1-2\lambda)(2v-v^2)}{2} > \frac{\lambda(v-v^2)}{2} + (1-2\lambda)(v/2 + 5/8) + \lambda v \\
\Leftrightarrow &\lambda > \lambda_1 := \frac{5 - 4v + 4v^2}{14 - 12v + 12v^2}
\end{aligned}$$

One can see that $\lambda_1$ and $\lambda_2$ increases in $v$. One can also show that there exists $\underline{v} < 7/25$ such that $\lambda_1 < \lambda_2$ when $\underline{v} < v < 1/2$. Therefore, (0,1/2) will never be an equilibrium if $\underline{v} < v < 1/2$. We assume that $\underline{v} < v < 1/2$ in subsequent analyses.
Now suppose the equilibrium is (1/2,1/2). One can see that the firm will deviate to 0 or 1 if $\lambda > \lambda_2$. Similarly, if the equilibrium is (0,1), the firm will deviate to 1/2 if $\lambda < \lambda_1$.
In sum, the equilibrium is (1/2,1/2) if $\lambda < \lambda_1$, (1/2,1/2) or (0,1) if $\lambda_1 < \lambda < \lambda_2$, and (0,1) if $\lambda > \lambda_2$.

Now consider the equilibrium under contextual ad targeting.
We first characterize the equilibrium ad choices and profits for each firm.

A. Under (1/2,1/2) strategy profile
   By symmetry, $a = \mathbf{E}(\theta) = 1/2$. $\mathbf{Var}(\theta) = \lambda v(2-v)/[3 + 4v - 4v^2 + \lambda(4v^2 - 6)]$.

$$\begin{aligned}
\pi(1/2, 1/2) =& D(1/2, 1/2) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\
=& \lambda(2v - v^2) + \frac{(1-2\lambda)[1-(1/2-v)^2]}{2} - \frac{\gamma\lambda(2v-v^2)}{4}
\end{aligned}$$

B. Under (0,1) strategy profile
   By symmetry, we only need to consider firm 1. Firm 2's ad choice will be $1 - a_1$, and firm 2's profit will be identical to firm 1's.
   $\mathbf{Var}(\theta) = \frac{1}{4} \frac{(1-2\lambda)(2v-v^2)/2}{(v+1/2)\lambda + (1-2\lambda)(2v-v^2)/2} \frac{(v+1/2)\lambda}{(v+1/2)\lambda + (1-2\lambda)(2v-v^2)/2}$.

$$\begin{aligned}
\pi(0, 1) =& D(0, 1) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\
=& (v + 1/2)\lambda + (1-2\lambda)(2v - v^2)/2 - \gamma \frac{1}{4} \frac{\frac{1}{2}(1-2\lambda)(2v-v^2)(v+1/2)\lambda}{(v+1/2)\lambda + (1-2\lambda)(2v-v^2)/2}
\end{aligned}$$

C. Under (0,1/2) strategy profile

Consider firm 1 first.

$$\mathbf{Var}_1(\theta) = \frac{1}{4}\frac{(1-2\lambda)(v-v^2)/2}{(v/2+5/8)\lambda+(1-2\lambda)(v-v^2)/2}\frac{(v/2+5/8)\lambda}{(v/2+5/8)\lambda+(1-2\lambda)(v-v^2)/2}.$$

$$\begin{aligned}\pi_1(0,1/2) =& D_1(0,1/2)\cdot[1-\gamma\mathbf{Var}_1(\theta)]\\ =&(\frac{v}{2}+\frac{5}{8})\lambda+\frac{(1-2\lambda)(v-v^2)}{2}-\gamma\frac{1}{4}\frac{\frac{1}{2}(1-2\lambda)(v-v^2)(v/2+5/8)\lambda}{(v/2+5/8)\lambda+(1-2\lambda)(v-v^2)/2}\end{aligned}$$

Consider firm 2 then.

$$\mathbf{Var}_2(\theta) = \frac{(1-2\lambda(v/2+5/8)}{D_2(0,1/2)}\frac{1}{4}+\frac{\lambda v}{D_2(0,1/2)}\cdot 1-[\frac{(1-2\lambda(v/2+5/8)}{D_2(0,1/2)}\frac{1}{2}+\frac{\lambda v}{D_2(0,1/2)}\cdot 1]^2.$$

$$\begin{aligned}\pi_2(0,1/2) =& D_2(0,1/2)\cdot[1-\gamma\mathbf{Var}_2(\theta)]\\ =&\frac{1}{8}[5+4v-2\lambda(5-2v+2v^2)][1+\frac{\gamma\lambda v[-15-7v+4v^2+6\lambda(5-3v+4v^2)]}{[5+4v-2\lambda(5-2v+2v^2)]^2}].\end{aligned}$$

Suppose the equilibrium is (0,1/2). One can show that there exists a $\gamma^d$ such that firm 1 will deviate from $x=0$ to $x=1/2$ if and only if:

$$\pi(1/2,1/2) > \pi_1(0,1/2) \Leftrightarrow \gamma < \gamma^d$$

Moreover, $\gamma^d$ increases in $v$ for $v \in (0,1/2)$ and $\lambda \in (1/3,1/2)$. Given that $1/3<\lambda<1/2$ and $0<v<1/2$, we have $\gamma^d>0 \Leftrightarrow (3-\sqrt{5})/4<v<1/2$ and $1/3<\lambda<\lambda_2$. An immediate implication is that this deviation never happens if $\lambda \geq \lambda_2$.
One can show that there exists a $\gamma^{d'}$ such that firm 2 will deviate from $x=1/2$ to $x=1$ if and only if:[3]

$$\pi(0,1) > \pi_2(0,1/2) \Leftrightarrow \gamma > \gamma^{d'}$$

Moreover, given that $1/3<\lambda<1/2$ and $0<v<1/2$, we have $\gamma^{d'} \leq 0 \Leftrightarrow \lambda_1 \leq \lambda < 1/2$. An immediate implication is that deviation always happens if $\lambda \geq \lambda_1$. So, (0,1/2) may only be an equilibrium if $\lambda<\lambda_1$. Now suppose $\lambda<\lambda_1$. In this case, we have shown that firm 1 will deviate if $\gamma<\gamma^d$ and firm 2 will deviate if $\gamma>\gamma^{d'}$.
Now suppose the equilibrium is (1/2,1/2). One can see that the firm will deviate to 0 or 1 if $\gamma>\gamma^d$, which may hold if $\lambda<\lambda_2$ and always holds if $\lambda>\lambda_2$. Similarly, if the equilibrium is (0,1), the firm will deviate to 1/2 if $\gamma<\gamma^{d'}$, which may hold if

[3] Firm 2 has a stronger incentive to deviate to $x=1$ rather than $x=0$ to soften competition. So, we only need to consider its deviation to $x=1$.

$\lambda < \lambda_1$ and never holds if $\lambda > \lambda_1$.

Since $\gamma^m > \max\{\gamma^d, \gamma^{d'}\}$, $\forall v \in (\underline{v}, 1/2), \lambda \in (1/3, 1/2)$, Let us define $\gamma^{d''}$ by $\min\{\gamma^d, \gamma^{d'}\}\mathbf{1}_{[\lambda<\lambda_1]} + \gamma^d \mathbf{1}_{[\lambda_1 \leq \lambda < \lambda_2]}$. We have that the equilibrium under contextual ad targeting is more polarized than that under behavioral ad targeting if $\gamma > \gamma^{d''}$ and $1/3 < \lambda < \lambda_2$. Moreover, $\gamma^{d''} < \gamma^m$ for all $\lambda \in (1/3, \lambda_2)$.

2. The base utility $v \geq 1/2$ : In this case, the consumer always derives a non-negative utility from consuming the content if the content location perfectly matches the consumer type. In addition, the consumer may consume the content even if the content location is on the opposite end of the consumer's location. One can show that $\overline{v} \geq 5/6$. The analyses are similar to the previous case.

■

*Proof of Proposition 8.* Based on the analyses of the main model, we need to show here that the additional utility term and the strategic choice of the consumers do not change the relative size of $\mathbf{Var}(\theta|x=0)$ and $\mathbf{Var}(\theta|x=1/2)$. We consider $M < 1/2$.

1. Suppose $x = 0$. Consider type $\theta = 0$ consumer first. We have $U(x, 0) \geq 1/2 - M + \epsilon$. Hence,

$$\begin{aligned}
&P(\text{a type 0 consumer consumes the content}) \\
\geq\ &P(1/2 - M + \epsilon \geq 0) \\
=\ &P(\epsilon \geq M - 1/2) \\
=\ &1 - F(M - 1/2) \\
=\ &1 - M
\end{aligned}$$

Now consider type $\theta = 1/2$ consumer.

$$\begin{aligned}
&P(\text{a type 1/2 consumer consumes the content}) \\
\leq\ &P(1/2 - 1/2 - 0 + \epsilon \geq 0) \\
=\ &1/2 \\
&P(\text{a type 1/2 consumer consumes the content}) \\
\geq\ &P(1/2 - 1/2 - M + \epsilon \geq 0) \\
=\ &1/2 - M
\end{aligned}$$

Therefore,

$$\mathbf{Var}(\theta|x = 0, M)$$

$$
\begin{aligned}
&= \mathbf{E}[\theta^2|x=0,M] - \mathbf{E}[\theta|x=0,M]^2 \\
&= \frac{1}{4}\frac{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)}{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)+P(\text{a type } 0 \text{ consumer consumes})\lambda} - \\
&\left[\frac{1}{2}\frac{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)}{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)+P(\text{a type } 0 \text{ consumer consumes})\lambda}\right]^2 \\
&= \frac{1}{4}\frac{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)}{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)+P(\text{a type } 0 \text{ consumer consumes})\lambda} \cdot \\
&\frac{P(\text{a type } 0 \text{ consumer consumes})\lambda}{P(\text{a type } 1/2 \text{ consumer consumes})(1-2\lambda)+P(\text{a type } 0 \text{ consumer consumes})\lambda} \\
&\leq \frac{1}{4}\frac{\frac{1}{2}(1-2\lambda)}{\frac{1}{2}(1-2\lambda)+(1-M)\lambda}\frac{\lambda}{(\frac{1}{2}-M)(1-2\lambda)+\lambda} \\
&\overset{M\to 0}{\to} \frac{1}{4}\frac{\frac{1}{2}(1-2\lambda)\lambda}{[\lambda+\frac{1}{2}(1-2\lambda)]^2} \\
&= \mathbf{Var}(\theta|x=0,M=0),
\end{aligned}
$$

which is the conditional variance in the base model.

2. Suppose $x = 1/2$. Similarly, we can show that in this case, $\mathbf{Var}(\theta|x = 1/2, M)$ is greater than or equal to an expression that converges to the conditional variance in the base model as $M \to 0$. According to Lemma 3, $\mathbf{Var}(\theta|x = 1/2) > \mathbf{Var}(\theta|x = 0)$ in the base model. Therefore, there exists $\hat{M} > 0$ such that $\mathbf{Var}(\theta|x = 1/2, M) > \mathbf{Var}(\theta|x = 0, M)$, for any $M \leq \hat{M}$. In such cases, the consumer's choice of niche content is privacy-reducing over the choice of mainstream content by definition. The same arguments as the ones in the main model imply that all the main insights of the monopoly case hold qualitatively if $M \leq \hat{M}$.

■

*Proof of Proposition 9.* We first show that the equilibrium number of firms located at $x = 0, N_0$ equals the equilibrium number of firms located at $x = 1, N_1$. Denote the equilibrium number of firms located at $x = 1/2$ by $N_{1/2}$. One can see that the expected ad revenues for firms located in the same position are the same. Denoted the expected ad revenue for a firm located at $x$ as $r(x)$.

Suppose that $N_0 \neq N_1$. Without loss of generality, we assume that $N_0 > N_1 > 0$. It implies that each of the $N_1$ firms located at $x = 1$ obtains a zero profit, $r(1) - c = 0$. One can see that $r(0) \cdot N_0 = r(1) \cdot N_1$, which implies $r(0) < r(1)$. But then, we have $r(0) - c < r(1) - c = 0$. It will not be an equilibrium because a firm located at $x = 0$ would earn a negative profit and would deviate by exiting the market. Consequently, $N_0 = N_1$ in equilibrium.

Now we compare the equilibrium $\mathbf{Var}(\theta|x = 0)$ with $\mathbf{Var}(\theta|x = 1/2)$.

$$P(x = 0|\theta = 0) = P(1/2 + \epsilon_0 > 0 + \epsilon_{1/2})$$

$$
\begin{aligned}
&= P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&= 1 - F(0) + \int_{-1/2}^{0} \int_{-1/2}^{\epsilon_0+1/2} f(\epsilon_{1/2}) d\epsilon_{1/2} f(\epsilon_0) d\epsilon_0 \\
&= 1 - F(0) + \int_{-1/2}^{0} F(\epsilon_0 + 1/2) f(\epsilon_0) d\epsilon_0 \\
&= 7/8, \\
P(x = 0|\theta = 1/2) &= P(0 + \epsilon_0 > 1/2 + \epsilon_{1/2}) \\
&= P(\epsilon_0 > \epsilon_{1/2} + 1/2) \\
&\overset{symmetry}{=} P(\epsilon_{1/2} > \epsilon_0 + 1/2) \\
&= 1 - P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&\overset{previous\ case}{=} 1 - 7/8 \ \ = 1/8
\end{aligned}
$$

$$
\begin{aligned}
&Privacy(\{0\}) \\
=&\mathbf{Var}[\theta|x = 0] \\
=&\mathbf{E}[\theta^2|x = 0] - \mathbf{E}[\theta|x = 0]^2 \\
=&(\frac{1}{2})^2 \cdot P(\theta = 1/2|x = 0) + 1^2 \cdot P(\theta = 1|x = 0) - \left\{\frac{1}{2} \cdot P(\theta = 1/2|x = 0) + 1 \cdot P(\theta = 1|x = 0)\right\}^2 \\
=&\frac{1}{4} \cdot \frac{P(x = 0|\theta = 1/2)P(\theta = 1/2)}{P(x = 0|\theta = 1/2)P(\theta = 1/2) + P(x = 0|\theta = 0)P(\theta = 0)} + 1 \cdot 0 - \\
&\left\{\frac{1}{2} \cdot \frac{P(x = 0|\theta = 1/2)P(\theta = 1/2)}{P(x = 0|\theta = 1/2)P(\theta = 1/2) + P(x = 0|\theta = 0)P(\theta = 0)} + 1 \cdot 0\right\}^2 \\
=&\frac{1}{4} \cdot \frac{\frac{1}{8}(1 - 2\lambda)}{\frac{1}{8}(1 - 2\lambda) + \frac{7}{8}\lambda} - \frac{1}{4} \cdot \left[\frac{\frac{1}{8}(1 - 2\lambda)}{\frac{1}{8}(1 - 2\lambda) + \frac{7}{8}\lambda}\right]^2 \\
=&\frac{1}{4} \frac{(1 - 2\lambda)(2 + 3\lambda)}{(1 + 5\lambda^2)}
\end{aligned}
$$

$$
\begin{aligned}
P(x = 1/2|\theta = 0) &= P(0 + \epsilon_{1/2} > 1/2 + \epsilon_0) \\
&= P(x = 0|\theta = 1/2) \\
&= 1/8 \\
P(x = 1/2|\theta = 1/2) &= P(1/2 + \epsilon_{1/2} > 0 + \epsilon_0 \ \& \ 1/2 + \epsilon_{1/2} > 0 + \epsilon_1) \\
&= P(1/2 + \epsilon_{1/2} > \max\{\epsilon_0, \epsilon_1\}) \\
&= P(\epsilon_{1/2} \geq 0) \cdot 1 + P(\epsilon_{1/2} < 0 \ \& \ 1/2 + \epsilon_{1/2} > \max\{\epsilon_0, \epsilon_1\})
\end{aligned}
$$

$$= 1/2 + \int_{-1/2}^{0} \int_{-1/2}^{1/2+\epsilon_{1/2}} 2(m+1/2) \cdot 1 \; dm \; d\epsilon_{1/2}$$
$$= 19/24,$$

$$\begin{aligned}
\text{where the cdf of } \max\{\epsilon_0, \epsilon_1\},\ F_m(m) &= P(\max\{\epsilon_0, \epsilon_1\} \leq m) \\
&= P(\epsilon_0 \leq m \ \&\ \epsilon_1 \leq m) \\
&\overset{independence}{=} P(\epsilon_0 \leq m) P(\epsilon_1 \leq m) \\
&= (m+1/2)^2,\ \forall m \in (-1/2, 1/2), \\
\text{so, the pdf of } \max\{\epsilon_0, \epsilon_1\},\ f_m(m) &= F_m'(m) = 2(m+1/2),\ \forall m \in (-1/2, 1/2).
\end{aligned}$$

$$\begin{aligned}
&Privacy(\{1/2\}) \\
=&\mathbf{Var}[\theta | x = 1/2] \\
=&\mathbf{E}[(\theta - 1/2)^2 | x = 1/2] \\
=&(1/2)^2 \cdot P(\theta = 0 | x = 1/2) + \cdot (1/2)^2 \cdot P(\theta = 1 | x = 1/2) \\
=&2 \cdot (1/2)^2 \cdot P(\theta = 0 | x = 1/2) \\
=&\frac{1}{2} \cdot \frac{P(x = 1/2 | \theta = 0) P(\theta = 0)}{P(x = 1/2 | \theta = 0) P(\theta = 0) + P(x = 1/2 | \theta = 1) P(\theta = 1) + P(x = 1/2 | \theta = 1/2) P(\theta = 1/2)} \\
=&\frac{1}{2} \cdot \frac{P(x = 1/2 | \theta = 0) P(\theta = 0)}{2 P(x = 1/2 | \theta = 0) P(\theta = 0) + P(x = 1/2 | \theta = 1/2) P(\theta = 1/2)} \\
=&\frac{1}{2} \cdot \frac{\frac{1}{8}\lambda}{2 \cdot \frac{1}{8}\lambda + \frac{5}{6}(1 - 2\lambda)} \\
=&\frac{3\lambda}{38 - 64\lambda}
\end{aligned}$$

$$\begin{aligned}
&Privacy(\{0\}) < Privacy(\{1/2\}) \\
\Leftrightarrow &\frac{1}{4} \frac{(1 - 2\lambda)(2 + 3\lambda)}{(1 + 5\lambda^2)} < \frac{3\lambda}{38 - 64\lambda} \\
\Leftrightarrow &g(\lambda) := -84\lambda^3 + 284\lambda^2 + 178\lambda - 76 > 0 \qquad (10)
\end{aligned}$$

Note that $g'(\lambda) = -252\lambda^2 + 568\lambda + 178,\ g''(\lambda) = -504\lambda + 568$. One can see that $g''(\lambda) > 0$ because $\lambda < 1/2$. Since $g'(1/3) > 0$, $g'(\lambda) > 0,\ \forall \lambda \in (1/3, 1/2)$. Since $g(1/3) > 0$, $g(\lambda) > 0,\ \forall \lambda \in (1/3, 1/2)$. Condition (10) always holds. Therefore, $Privacy(\{0\}) < Privacy(\{1/2\})$: the consumer's choice of niche content is privacy-reducing over the choice of mainstream content.

We now show that the ratio of the number of niche firms to the number of mainstream firms is higher under contextual ad targeting than under behavioral ad targeting.

Denote the number of niche firms located at $x = 0$ under behavioral ad targeting by $N_0^b$, the number of mainstream firms located at $x = 1/2$ under behavioral ad targeting by $N_{1/2}^b$, the number of niche firms located at $x = 0$ under contextual ad targeting by $N_0^c$, and the number of mainstream firms located at $x = 1/2$ under contextual ad targeting by $N_{1/2}^c$.

The market clearing condition implies that in equilibrium,

$$\begin{cases} D(0)/N_0^b - c = 0 \\ D(1/2)/N_{1/2}^b - c = 0 \\ D(0)[1 - \gamma Privacy\{0\}]/N_0^c - c = 0 \\ D(1/2)[1 - \gamma Privacy\{1/2\}]/N_{1/2}^b - c = 0 \end{cases}$$

$\Rightarrow N_0^c/N_{1/2}^c = N_0^b/N_{1/2}^b \cdot \frac{1-\gamma Privacy\{0\}}{1-\gamma Privacy\{1/2\}} > N_0^b/N_{1/2}^b$. ■

# 3 Extension with $K > 2$ Firms

**Lemma 5.** *Suppose there are $M$ mainstream firms and $N_0$ and $N_1$ niche firms at ideological positions $x = 0$ and $x = 1$, respectively. The probability of a type-$\theta$ consumer choosing a media firm at position $x$ is presented in the table below:*

| *Conditional Probability* | *Value* |
|---|---|
| $P(x = 0 \mid \theta = 0)$ | $\mathbb{1}(N_0 > 0)\left[1 - \mathbb{1}(M > 0)(1/2)^{N_0+M}\left(\sum_{k=1}^{M}\binom{M}{k}\frac{k}{N_0+k}\right)\right]$ |
| $P(x = 1/2 \mid \theta = 0)$ | $\mathbb{1}(M > 0)(1/2)^{N_0+M}\left(\sum_{k=1}^{M}\binom{M}{k}\frac{k}{N_0+k}\right)$ |
| $P(x = 1 \mid \theta = 0)$ | *0* |
| $P(x = 0 \mid \theta = 1/2)$ | $\mathbb{1}(N_0 > 0)\frac{N_0}{N_0+N_1}(1/2)^{N_0+M+N_1}\left(\sum_{k=1}^{N_0+N_1}\binom{N_0+N_1}{k}\frac{k}{M+k}\right)$ |
| $P(x = 1/2 \mid \theta = 1/2)$ | $\mathbb{1}(M > 0)\left[1 - \mathbb{1}(N_0 + N_1 > 0)(1/2)^{N_0+M+N_1}\left(\sum_{k=1}^{N_0+N_1}\binom{N_0+N_1}{k}\frac{k}{M+k}\right)\right]$ |
| $P(x = 1 \mid \theta = 1/2)$ | $\mathbb{1}(N_1 > 0)\frac{N_1}{N_0+N_1}(1/2)^{N_0+M+N_1}\left(\sum_{k=1}^{N_0+N_1}\binom{N_0+N_1}{k}\frac{k}{M+k}\right)$ |
| $P(x = 0 \mid \theta = 1)$ | *0* |
| $P(x = 1/2 \mid \theta = 1)$ | $\mathbb{1}(M > 0)(1/2)^{N_1+M}\left(\sum_{k=1}^{M}\binom{M}{k}\frac{k}{N_1+k}\right)$ |
| $P(x = 1 \mid \theta = 1)$ | $\mathbb{1}(N_0 > 0)\left(1 - \mathbb{1}(M > 0)(1/2)^{N_1+M}\left(\sum_{k=1}^{M}\binom{M}{k}\frac{k}{N_1+k}\right)\right)$ |

*Proof.* Let $\epsilon_j^{(x)}$ denote the idiosyncratic term for the $j^{th}$ firm at position $x$. We know that all idiosyncratic terms are independently drawn from $U[-1/2, 1/2]$.

We consider all three cases for $\theta$.

1. *Case* $\theta = 0$: We know that a consumer with $\theta = 0$ will never choose $x = 1$, because $U(x = 1; \theta = 0) < 0$. So the choice is between $N_0$ niche firms at position $x = 0$ and $M$ mainstream firms. The consumer receives utility $1/2 + \epsilon_j^{(0)}$ from choosing the firm at $x = 0$, and utility

$\epsilon_i^{(1/2)}$ from choosing the firm at $x = 1/2$. As such, the consumer chooses a mainstream firm if there is at least one element in $\{\epsilon_i^{(1/2)}\}_{i=1}^{M}$ is greater than all elements in $\{1/2 + \epsilon_j^{(0)}\}_{j=1}^{N_0}$. We know that if any $\epsilon_j^{(0)} > 0$, there is a zero probability of a mainstream media firm being chosen. So for a mainstream firm to be chosen, we require all $\epsilon_j^{(0)}$'s to come from $[-1/2, 0]$, which has a probability of $(1/2)^{N_0}$. In that event, the mainstream firm $k$ has a chance to be chosen by the consumer only if $\epsilon_i^{(1/2)} \sim U[0, 1/2]$, i.e., the upper half of the uniform distribution. For any event where the error $k$ out of $M$ firms come from $U[0, 1/2]$ (i.e., $M - k$ firms with error from $U[-1/2, 0]$), the probability of one of these $k$ values being the highest is $k/(k + N_0)$. We can characterize all such events with a Binomial distribution $B(M, 1/2)$, where the success probability refers to the error term being drawn from the upper half of $U[-1/2, 1/2]$. Therefore, we can write:

$$
\begin{aligned}
P(x = 1/2 \mid \theta = 0) &= (1/2)^{N_0} \left( \sum_{k=1}^{M} \binom{M}{k} (1/2)^k (1/2)^{M-k} \frac{k}{N_0 + k} \right) \\
&= (1/2)^{N_0+M} \left( \sum_{k=1}^{M} \binom{M}{k} \frac{k}{N_0 + k} \right)
\end{aligned}
$$

Based on the probability above, we can define the following probability:

$$
P(x = 0 \mid \theta = 0) = 1 - (1/2)^{N_0+M} \left( \sum_{k=1}^{M} \binom{M}{k} \frac{k}{N_0 + k} \right)
$$

2. *Case* $\theta = 1$: This case is conceptually the same as $\theta = 0$, but the difference is in the number of firms at position $x = 1$. We can use the logic presented in the case above to arrive at the following conditional probabilities:

$$
\begin{aligned}
P(x = 0 \mid \theta = 1) &= 0 \\
P(x = 1/2 \mid \theta = 1) &= (1/2)^{N_1+M} \left( \sum_{k=1}^{M} \binom{M}{k} \frac{k}{N_1 + k} \right) \\
P(x = 1 \mid \theta = 1) &= 1 - (1/2)^{N_1+M} \left( \sum_{k=1}^{M} \binom{M}{k} \frac{k}{N_1 + k} \right)
\end{aligned}
$$

3. *Case* $\theta = 1/2$: In this case, it is possible that the consumer chooses any niche type. The consumer receives utility $1/2 + \epsilon_i^{(1/2)}$ from choosing any mainstream firm. On the other hand, the consumer receives utility $\epsilon_j^{(0)}$ or $\epsilon_j^{(1)}$ from choosing firms at each niche position. Using the logic from previous cases, we can write the following probability for the union of events

where consumer of type $\theta = 1/2$ chooses either the firm at $x = 0$ or $x = 1$:

$$P(x = 0 \vee x = 1 \mid \theta = 1/2) = (1/2)^M \left( \sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} (1/2)^k (1/2)^{N_0+N_1-k} \frac{k}{M+k} \right)$$
$$= (1/2)^{N_0+M+N_1} \left( \sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} \frac{k}{M+k} \right)$$

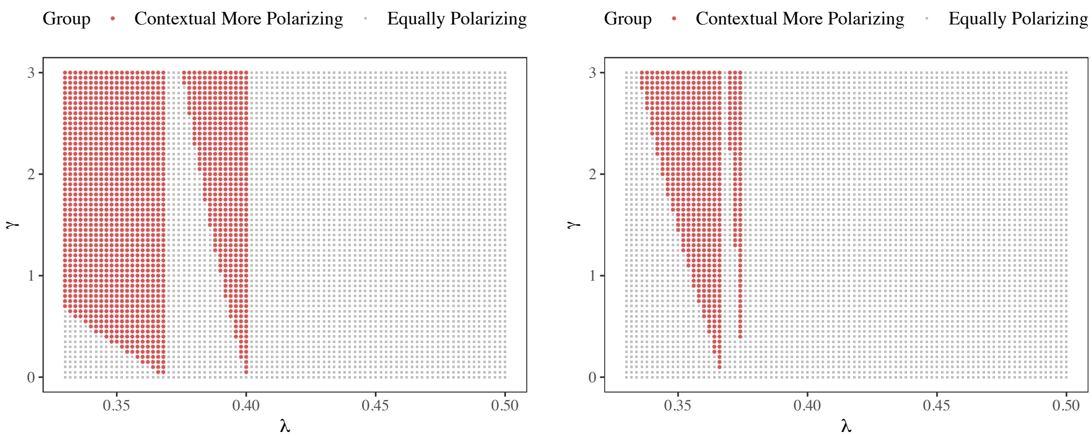
The probability above is for the event where the consumer either chooses $x = 0$ or $x = 1$. Since the errors are independently drawn, the probability of choosing each niche position is proportional to the number of firms in that niche position. Hence, we can write the following probabilities for all three events:

$$P(x = 0 \mid \theta = 1/2) = \frac{N_0}{N_0 + N_1} (1/2)^{N_0+M+N_1} \left( \sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} \frac{k}{M+k} \right)$$
$$P(x = 1 \mid \theta = 1/2) = \frac{N_1}{N_0 + N_1} (1/2)^{N_0+M+N_1} \left( \sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} \frac{k}{M+k} \right)$$
$$P(x = 1/2 \mid \theta = 1/2) = 1 - (1/2)^{N_0+M+N_1} \left( \sum_{k=1}^{N_0+N_1} \binom{N_0 + N_1}{k} \frac{k}{M+k} \right)$$

■

We now use the results from Lemma 5 to examine the equilibrium for cases with $K > 2$ media firms. Since we have a closed-form expression for conditional content choice probabilities, we can compute demand, conditional variance, and the total profit for each player in a $(N_0, M, N_1)$ strategy profile under both contextual and behavioral targeting for a fixed set of $\lambda$ and $\gamma$. This means that for a fixed set of $\lambda$ and $\gamma$, we can enumerate over all $(N_0, M, N_1)$ and assess if that strategy profile is an equilibrium by checking whether a player has an incentive to deviate. We know that the total number of profiles such that $N_0 + M + N_1 = K$ is equal to $\binom{K+2}{2}$, highlighting that the computational complexity of finding equilibria grows polynomially in the number of firms for fixed values of $\lambda$ and $\gamma$.

We adopt this computational approach for $K = 3$ and $K = 4$ cases, and use a grid of 86 $\lambda$ values in $[0.33, 0.50]$ with a 0.002 precision, and 61 $\gamma$ values in $[0, 3]$ with a 0.05 precision. Figure 6 shows the cases in the parameter space where the equilibrium under contextual ad targeting is more, equally, and less polarizing than the equilibrium under behavioral ad targeting, for $K = 3$ (in Figure 6a) and for $K = 4$ (in Figure 6b). We call an equilibrium more polarizing if there is a higher proportion of firms in niche positions. Red points in these figures show the regions where the equilibrium is more polarizing under contextual ad targeting compared to behavioral ad targeting. This is the same as shaded region in Figure 5. As shown in both Figures 6a and 6b, our qualitative insight holds in these cases: under all possible combinations of $(\lambda, \gamma)$ in our grid search, the equilibrium under contextual ad targeting is at least as polarizing as the equilibrium under behavioral ad targeting.




(a) Case with 3 Firms ($K = 3$)

(b) Case with 4 Firms ($K = 4$)

Figure 6: Comparison of equilibrium under contextual and behavioral ad targeting in cases with more than two firms when $\epsilon \sim U[-1/2, 1/2]$.

*Note*: Points in gray refer to cases where the equilibrium is equally polarizing under behavioral and contextual ad targeting. Red stars refer to cases where the equilibrium under contextual ad targeting is more polarizing that that under behavioral ad targeting (i.e., higher proportion of niche firms).